

# Quantum Decoherence, Entropy and Thermalization in Strong Interactions at High Energy <sup>1</sup>

## I. Noisy and Dissipative Vacuum Effects in Toy Models

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### Abstract

Entropy is generated in high-multiplicity events by a dynamical separation of strongly interacting systems into partons and unobservable environment modes (almost constant field configurations) due to confinement. The effect is demonstrated in a non-relativistic single-particle model and a scalar field theory, where it amounts to quantum field Brownian motion. We analyze the quantum decoherence of partons, which formally corresponds to non-unitary time-evolution and causes entropy production, in terms of Schmidt and pointer states in the non-relativistic case. For the coupled scalar fields (partons and environment) we derive the Cornwall-Jackiw-Tomboulis effective action and equations of motion with the non-perturbative time-dependent variational method in TDHF approximation (to study the time-evolution of model structure functions in the sequel). We obtain a model-independent lower bound for the entropy in terms of two-point Wightman functions.

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## 1. Introduction

The idea of highly excited hadronic matter can be traced back to the earliest observations of high-energy cosmic ray events with an associated high multiplicity of secondary produced hadrons. Most notably Fermi and Landau in the early 1950's initiated a statistical approach coupled with a hydrodynamical description of the high energy density matter, which is formed and subsequently evolves in such prototype strong interactions [1, 2] (see also [3] for a personal historical account and further references). Until the present day their pioneering work has influenced further developments in this field, even if for about two decades there existed no hint whatsoever to the most relevant if not fundamental degrees of freedom of high energy density matter.

The statistical approach, in particular, is motivated by considering the large amount of initial kinetic energy which is carried into the reaction by the colliding particles and is ultimately transformed into the large multiplicity of secondary particles, i.e. produced matter which finally flies apart. By a fast (on the scale of fm/c) compression of a relativistically large amount of energy ( $\gg 1\text{GeV/nucleon}$ ) into a small volume (on the scale of fm<sup>3</sup>) one couples very effectively to strongly interacting degrees of freedom, i.e. those which materialize as the observed hadronic particles. Loosely speaking, a large number of initially hidden underlying field degrees of freedom must have been excited during this transformation of the apparently simple (ordered) initial state into the obviously rather complicated (disordered) final state of the collision: It seems as if an enormous amount of “entropy” can be “created” here, which experimentally increases with energy and complexity of the colliding hadrons or nuclei. Where does this entropy come from which is one of the characteristic features of multiparticle production events? How can it be interpreted microscopically in terms of a field theory of strong interactions?

Further on, working towards a statistical theory of high energy density matter, Hagedorn led the important intermediate step of taking into account the information about the dynamics available in the 1960's in the statistical bootstrap model in the form of an exponential hadronic mass spectrum [4, 5]. Here, as with all former and later models and their phenomenologically rather successful applications, the statistical approach immediately suggests the crucially simplifying assumption of local thermal (kinetic and possibly chemical) equilibrium among the relevant constituents of matter in very inelastic high-energy reactions. After the advent of QCD the correct theory of strong interactions is nowadays believed to be firmly established with quark and gluon fields as the fundamental degrees of freedom. Thus, corroborated by the observation that asymptotically at short distances the running coupling of QCD vanishes and once again guided by the equilibrium assumption, it has been an ongoing and ever stronger effort to identify theoretically unambiguous characteristics and find experimentally signals of a hot and dense “quark-gluon plasma” of almost non-interacting quarks and gluons. This intermediate new state of matter should be formed at sufficiently high energy density during hadronic or nuclear collisions (see references [4] and [6], respectively, for early and latest overviews of this field). Until

very recently, however, the equilibrium assumption (“thermalization”) has been completely ad hoc and is still lacking any detailed justification or deeper understanding. Even if many global features of high-energy reactions can be reproduced in thermal models of varying degree of sophistication, it has remained quite a mystery, why they work so well.

Connected to the problem of large entropy production mentioned above, the questions arising in the context of thermalization are: Why does the thermalization, which implies a saturation of entropy production, proceed so fast (on the scale of fm/c)? Which are the relevant degrees of freedom in terms of QCD and with what kind of effective interactions (“hard” perturbative vs. “soft” non-perturbative physics) that contribute most efficiently to thermalization? How do these “randomizing” degrees of freedom hide again during the final hadronization process?

Questions pertaining directly to the latter hadronization or confinement problem presumably will have to stay off-stage for a while. However, even those concerning (time scales of) entropy production and thermalization cannot yet be seriously addressed. As we explain in this paper, the necessary quantitative studies have not been clearly defined from a conceptual point of view. It is our purpose here to present an analysis of the entropy production in very inelastic high-energy reactions with an eye towards the parton picture, to provide a plausible hypothesis about the underlying randomization process, and to outline some calculations under simplifying assumptions, which should strengthen the proposed scheme.

The plan of our paper is as follows. In Sec. 2 we introduce the definitions and formal apparatus which will be applied to analyze the entropy problem. We employ density matrices and, especially, we explain the notions of *Schmidt and pointer states in the Hilbert space of a complex system* and of *environment-induced quantum decoherence*. They play an important role in studies of the measurement process and the classical limit of quantum theory (Ref. [7] provides a useful introduction with numerous references to original work; see also the review [8]). Recent attempts to formulate a consistent approach to quantum cosmology initiated by Gell-Mann and Hartle and others have given a new impetus to study these questions again [7, 9], which touch upon the foundations of quantum theory. Basically, there one tries to understand, why the universe considered as a closed quantum system (in an overall pure state) looks as classical as it does.

For example, low-energy propagating modes could be coupled to dislocalized discrete modes left over from a topological phase in string theory, which latter ones are hidden from a low-energy observer performing “measurements” by local scattering methods [10]. Thus, there is a dynamical separation of modes according to whether they are observable in the low-energy limit of string theory or not. The unobservable modes naturally constitute an environment which may induce quantum decoherence in the subsystem of the observable modes. This in turn leads to *non-equilibrium evolution*, i.e. non-unitary quantum mechanics in the subsystem with an associated direction of time and entropy production, in particular.

Analogously we may ask, why a pure-state high-energy scattering experiment lends itself to a statistical description characterized by a large apparent entropy and possibly quasi-classical propagation of partons. In this more down-to-earth context of

strong interactions similar phenomena should occur on a microscopical scale ( $\Lambda_{QCD} \approx 200\text{MeV}$ ) which can be related to a *dynamical separation* of a complex system into an *observable subsystem* and an *unobservable environment*. Surprisingly, the subject of our present work has not been analyzed from this appealing point of view up to now. The paradigm of *confinement* has been around for about twenty years and it seems unavoidable by now to think of unobservable quark and gluon states or, rather, long-wavelength field modes feeling or producing confinement as constituting a *dynamically active environment leading to quantum decoherence of the “almost observable” partons* (in the sense of parton-hadron duality or deep-inelastic scattering), to which we have become used. Thus, the situation is opposite to the one in string theory; in QCD almost constant low-energy field modes constitute the environment for the “hard” partons.

This may have important consequences for the parton picture as applied to nuclear collisions or complex reactions, in general, where factorization and assumptions about the quasi-classical propagation of partons between successive hard scatterings come into play [11, 12]. The so-called “sudden approximation” underlying the parton model, which can consistently be justified *within* perturbation theory, could fail or, rather, be justified on a deeper level precisely if and when the strongly coupled environment becomes important. After all, the commonly referred to analogy between a parton and a hard-struck electron in a crystal can be rather misleading, since only in the latter case *all* relevant interactions are weak.

Recently there have been speculations that so far unexplained decohering effects on the initial state multi-parton wave functions of colliding hadrons or nuclei should contribute considerably to entropy production in these reactions, see for example [13, 14]. We show that the *mechanism for entropy production* in strong interactions at high energy is hidden in the active role of the environment.

In Sec. 3 we present a non-relativistic toy model of a parton linearly coupled to gluonic oscillators which serves to illustrate the formalism of Sec. 2 with environment-induced quantum decoherence and entropy production in the partonic subsystem. This model bears some resemblance to a non-relativistic electron interacting with the quantized electromagnetic field. The oscillator spectral density of the gluonic environment, however, is by construction rather different and receives its dominant contribution in the infrared, in particular. The encouraging results obtained in a short-time strong-coupling approximation employing the Feynman-Vernon influence functional technique will guide us in Sec. 4, where we begin to study the problem of strong interactions in the parton picture.

We propose a relativistic field theory generalizing the toy model of Sec. 3 consisting of two non-linearly but locally coupled scalar fields, which we assume to represent the partons and the confined (or confining vacuum) environment modes, respectively. The purpose of this model, which may still be far removed from a phenomenologically viable representation of QCD, is to tackle the particular technical problems expected in the field theory context. Loosely speaking one could also say that we extend here the study of *quantum Brownian motion* into the realm of field theory. As it turns out, a functional Schrödinger picture approach based on the time-dependent variational principle of Dirac seems very promising [15, 16]. It allows to transform the usual

Heisenberg operator approach to applications of quantum field theory, i.e. mainly time-independent scattering problems, into a set of manageable variational equations describing the relevant initial value problem. Furthermore, an important advantage of this procedure here is not to be limited to the study of weakly interacting systems, as it is the case with ordinary perturbation theory.

In a time-dependent Hartree-Fock type approximation we can calculate the entropy production for *any* scalar parton model field theory completely in terms of two-point Wightman or correlation functions, which seems to be a rather general result allowing interesting conclusions concerning the real QCD problem. In order to learn about the time-evolution of the two-point functions, of course, one has to specify the interactions in detail. Similar in spirit to the BCS theory of superconductivity, where it was realized that particularly in the strong-coupling limit the “action” is around the Fermi surface in a conductor [17], we propose here to distinguish between modes which are close to constant fields (*environment modes*) and those which are not (*partons*). Thus, a Momentum Space Mode Separation is the essential feature of our toy model field theory.

In a sequel to the present work we further evaluate the model of Sec. 4 and explicitly calculate the time-evolution of the parton density (functional) matrix for various physical initial conditions. It is important then to derive the relation with non-perturbative (spin-, colour-, and flavour-averaged) QCD parton structure functions [11, 12, 13], which are experimentally accessible. Furthermore, we will refer to the parton cascade approach for hadronic or nuclear collisions [18]. This model, which is based on perturbative QCD to the largest extent possible at present and singled out among existing “Monte Carlo” procedures by most clearly specifying the model assumptions about non-perturbative soft processes, is particularly useful as a reference frame in trying to understand essential aspects of entropy production in high-energy reactions. New effects implied by a dynamical treatment of the unobservable environment modes, as initiated here, have to be contrasted with such a state of the art description, either to better justify or to modify it where necessary. In particular, assumptions about the factorization of multiple hard or semi-hard parton scatterings, i.e. quantum mechanically independent rescattering [11, 12, 18], deserve a closer look.

Finally, we have to go over from a somewhat ad hoc model to full QCD. We propose an explanation of the underlying decoherence/randomization process in terms of unobservable soft gluon background fields, neglecting potentially important soft quark modes (e.g. chiral condensate) for simplicity. Presumably, this necessitates model assumptions about the properties of the background fields, since a non-perturbative ab initio treatment of QCD seems still out of reach. However, detailed and phenomenologically satisfactory approximations are available here, e.g. in the form of a “stochastic vacuum” model (see e.g. Refs. [19] and references therein). They can be implemented by suitably modifying the method of background field quantization (see e.g. [20] and references therein).

The studies and further remarks presented here should indicate the next steps to verify our hypothesis about the nature and importance of environment-induced quantum decoherence in strong interactions, its particular relation to confinement

and unobservable long-wavelength QCD fields, and the ensuing solution of the long-standing “entropy puzzle” in high-energy reactions.

In the final Sec. 5 we discuss our main results. We also point out further interesting applications of the formalism developed presently. - Throughout this paper we work with units such that  $\hbar = c = k_B = 1$ .

## 2. Classical Behaviour and Entropy in Pure-state Quantum Systems

To begin with, we want to derive the *Schmidt decomposition* for the density matrix of a closed quantum system.<sup>3</sup> - We may think of colliding hadrons or nuclei which are sufficiently separated from any other strongly interacting matter as an approximate realisation of such a system (neglecting any long-range interactions) or the universe as a whole, which most fascinating example, however, is presently understood the least. - Thus, the complex system is described by a pure quantum state  $|\Psi\rangle$  which evolves according to some still to be specified dynamics in an embedding Hilbert space  $\mathcal{H}$ .

The corresponding density matrix is given by a hermitian operator,

$$\hat{\rho} = |\Psi\rangle\langle\Psi| , \quad (1)$$

which is trivially diagonal. Suppose now that we are experimentally limited or, rather, we are *forced by the dynamics of the complex physical system to observe only certain components of the complete state vector*  $|\Psi\rangle$ . I.e., we deliberately consider a factorization of the Hilbert space into two predetermined subspaces with orthonormal bases  $\mathcal{P} \equiv \{|\rangle\}$  (“partonic subsystem”) and  $\mathcal{G} \equiv \{| \rangle\}$  (“gluonic environment”), respectively.<sup>4</sup> Then, with  $\mathcal{H} = \mathcal{P} \otimes \mathcal{G}$ ,

$$|\Psi\rangle = \sum_{p,g} c_{pg} |p\rangle|g\rangle , \quad (2)$$

and,

$$\hat{\rho} = \sum_{p,g,p',g'} c_{pg} c_{p'g'}^* |p\rangle|g\rangle\langle g'|\langle p'| , \quad (3)$$

are corresponding general expansions. Furthermore, defining

$$|p_{\mathcal{G}}\rangle \equiv \sum_g c_{pg} |g\rangle , \quad (4)$$

we obtain

$$|\Psi\rangle = \sum_p |p\rangle|p_{\mathcal{G}}\rangle . \quad (5)$$

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<sup>3</sup>The derivation is given here to make our work selfcontained and to introduce necessary definitions and the notation; see also Ref. [21] for an earlier presentation in a different context.

<sup>4</sup>We choose these completely arbitrary labels in purpose to prepare the reader for Secs. 3 and 4 and will be interested particularly in the behaviour of the partonic subsystem later on.

The linear combinations  $|p_{\mathcal{G}}\rangle$  are not necessarily orthogonal or normalized. However, let us introduce the density operator for the partonic subsystem,

$$\hat{\rho}_{\mathcal{P}} \equiv \text{Tr}_{\mathcal{G}} \hat{\rho} = \sum_{p,g,p'} c_{pg} c_{p'g}^* |p\rangle\langle p'| , \quad (6)$$

by orthonormality of the  $\mathcal{G}$ -basis. Next, we assume that we chose the orthonormal basis of  $\mathcal{P}$  to be composed of the *eigenvectors* of  $\hat{\rho}_{\mathcal{P}}$ . Then, by its orthonormality we find

$$\text{const} \cdot \delta_{pp'} = \langle p | \hat{\rho}_{\mathcal{P}} | p' \rangle = \sum_g c_{pg} c_{p'g}^* = c_p \cdot \delta_{pp'} , \quad (7)$$

with a positive eigenvalue  $c_p \equiv \sum_g |c_{pg}|^2$ . Similarly, we obtain from eq. (4) using eq. (7)

$$\langle p'_{\mathcal{G}} | p_{\mathcal{G}} \rangle = \sum_g c_{pg} c_{p'g}^* = c_p \cdot \delta_{pp'} . \quad (8)$$

Thus, having chosen an eigenvector (of  $\hat{\rho}_{\mathcal{P}}$ ) basis of  $\mathcal{P}$ , the  $\{|p_{\mathcal{G}}\rangle\}$  *must be orthogonal* as well. Introducing the density operator of the gluonic environment and using eq. (4),

$$\hat{\rho}_{\mathcal{G}} \equiv \text{Tr}_{\mathcal{P}} \hat{\rho} = \sum_{g,p,g'} c_{pg} c_{pg'}^* |g\rangle\langle g'| = \sum_p |p_{\mathcal{G}}\rangle\langle p_{\mathcal{G}}| , \quad (9)$$

we also see that the  $|p_{\mathcal{G}}\rangle$ 's are the eigenvectors of  $\hat{\rho}_{\mathcal{G}}$  with eigenvalues  $\langle p_{\mathcal{G}} | p_{\mathcal{G}} \rangle = c_p$ , by eq. (8). Thus, surprisingly  $\hat{\rho}_{\mathcal{G}}$  and  $\hat{\rho}_{\mathcal{P}}$  have identical non-zero eigenvalues, the number of which is determined by the dimension of the smaller of the two subspaces  $\mathcal{P}, \mathcal{G} \subset \mathcal{H}$  or the rank of the matrix  $\{c_{pq}\}$  in eq. (2). Normalizing the  $|p_{\mathcal{G}}\rangle$ 's,

$$|p_{\mathcal{G}}^n\rangle \equiv c_p^{-1/2} |p_{\mathcal{G}}\rangle , \quad (10)$$

we obtain the Schmidt decomposition [22] of the complete state from eq. (5):

$$|\Psi\rangle = \sum_p c_p^{1/2} |p\rangle |p_{\mathcal{G}}^n\rangle , \quad (11)$$

in terms of orthonormal “*Schmidt states*”  $\{|p\rangle\}$ , which span  $\mathcal{P}$ , and  $\{|p_{\mathcal{G}}^n\rangle\} \subseteq \mathcal{G}$ . They represent the sets of *eigenvectors* of  $\hat{\rho}_{\mathcal{P}}$  and  $\hat{\rho}_{\mathcal{G}}$ , respectively. Of course, the roles of  $\mathcal{P}$  and  $\mathcal{G}$  can formally be exchanged in the construction. Finally, eqs. (6,7) yield

$$\hat{\rho}_{\mathcal{P}} = \sum_p c_p |p\rangle\langle p| , \quad (12)$$

and eqs. (9,10)

$$\hat{\rho}_{\mathcal{G}} = \sum_p c_p |p_{\mathcal{G}}^n\rangle\langle p_{\mathcal{G}}^n| . \quad (13)$$

Equations (11) - (13) are the essential results of the *Schmidt decomposition* procedure.

To appreciate the importance of the Schmidt decomposition, we proceed in several steps. First of all, we may generally assume a normalized complex system state  $|\Psi\rangle$ . This implies

$$1 = \langle \Psi | \Psi \rangle = \sum_p c_p = \text{Tr}_{\mathcal{P}} \hat{\rho}_{\mathcal{P}} = \text{Tr}_{\mathcal{G}} \hat{\rho}_{\mathcal{G}} , \quad (14)$$

where we used eqs. (11) - (13) and orthonormality of the Schmidt states. The sum rule (14) suggests to interpret the positive expansion coefficients  $c_p$  in eqs. (12,13) as *probabilities* to find e.g. the partonic subsystem in the respective states  $\{|p\rangle\}$  [23]. Such an interpretation is confirmed by calculating, for example, the probability  $P(\tilde{p})$  to find the partonic subsystem in a state  $|\tilde{p}\rangle \in \mathcal{P}$ ,

$$P(\tilde{p}) \equiv \langle \tilde{p} | \hat{\rho}_{\mathcal{P}} | \tilde{p} \rangle = \sum_p c_p |\langle p | \tilde{p} \rangle|^2 , \quad (15)$$

which is equal to  $c_{\tilde{p}}$  if  $|\tilde{p}\rangle$  is one of the Schmidt states. Or, we calculate the expectation value of an observable  $\hat{O}_{\mathcal{P}}$ ,

$$\langle \hat{O}_{\mathcal{P}} \rangle \equiv \text{Tr}_{\mathcal{P}} \hat{\rho}_{\mathcal{P}} \hat{O}_{\mathcal{P}} = \sum_p c_p \langle p | \hat{O}_{\mathcal{P}} | p \rangle , \quad (16)$$

which is the expectation value of  $\hat{O}_{\mathcal{P}}$  in a Schmidt state times the probability to find that state in the partonic subsystem summed over its whole basis.

Thus, we conclude that via the Schmidt decomposition the subsystem and its environment can be described by simultaneously diagonal mixed state density matrices ( $\hat{\rho}_{\mathcal{P},\mathcal{G}}^2 \neq \hat{\rho}_{\mathcal{P},\mathcal{G}}$ ). In particular, note the absence of quantum interference terms  $\propto \langle \tilde{p} | p \rangle \langle p' | \tilde{p} \rangle$  in eq. (15), for example, or  $\propto \langle p' | \hat{O}_{\mathcal{P}} | p \rangle$  in eq. (16), which represent *incoherent* sums. Hence, we achieved what is called *quantum decoherence* in the subsystem [7, 8]. Note that in distinction to eq. (16) we obtain

$$\langle \hat{O}_{\mathcal{G}} \rangle \equiv \text{Tr}_{\mathcal{G}} \hat{\rho}_{\mathcal{G}} \hat{O}_{\mathcal{G}} = \sum_{p,g} c_p \langle g | p_{\mathcal{G}}^n \rangle \langle p_{\mathcal{G}}^n | \hat{O}_{\mathcal{G}} | g \rangle , \quad (17)$$

where interference terms are present. In any case, the fact remains that the complex system *is* in the pure state  $|\Psi\rangle$ : decoherence is achieved in the subsystem at the expense of establishing unique *quantum correlations between subsystem and environment* [7, 21], which are explicitly constructed in eq. (11) above.

Next, focussing our interest on the partonic subsystem, we define its von Neumann or *statistical entropy* as usual [23],

$$S_{\mathcal{P}} \equiv -\text{Tr}_{\mathcal{P}} \hat{\rho}_{\mathcal{P}} \ln \hat{\rho}_{\mathcal{P}} = -\sum_p c_p \ln c_p > 0 , \quad (18)$$

recalling that  $\hat{\rho}_{\mathcal{P}}$  is diagonal with  $0 < c_p < 1$ , see eqs. (12,14), and assuming always at least two non-vanishing terms in the sum over states. Thus, a **non-zero entropy** emerges in a complex pure-state quantum system quite naturally, if only a subsystem is considered (or accessible to experiment) and the trace over the remaining environment degrees of freedom is calculated to obtain the relevant density submatrix. Note that the total entropy vanishes,  $S \equiv -\text{Tr} \hat{\rho} \ln \hat{\rho} = -1 \cdot \ln 1 = 0$ , as it should be. The same conclusions can be reached for the *linear entropy* employed in Refs. [24, 25],

$$S_{\mathcal{P}}^{\text{lin}} \equiv \text{Tr}_{\mathcal{P}} (\hat{\rho}_{\mathcal{P}} - \hat{\rho}_{\mathcal{P}}^2) = 1 - \text{Tr}_{\mathcal{P}} \hat{\rho}_{\mathcal{P}}^2 = 1 - \sum_p c_p^2 , \quad (19)$$



which more directly measures the “(im)purity” of a density matrix ( $0 < S_{\mathcal{P}}^{lin} < 1$ ) and is often easier to handle than the standard statistical entropy. In Sec. 4.2, eq. (114), we show that the linear entropy provides a *lower bound* on the statistical entropy.

Finally, the evolution of the complex system has to be taken into account. Up to now our considerations were limited to a fixed pure state and its Schmidt decomposition which, therefore, has to be recalculated from one instant to the next following the evolving system. Of course, the *unitary evolution* according to the Schrödinger equation,

$$i\partial_t |\Psi\rangle = \hat{H} |\Psi\rangle , \quad |\Psi\rangle = e^{-i\hat{H}t} |\Psi_0\rangle , \quad (20)$$

leaves the closed quantum system in a pure state, i.e. eq. (1) remains valid with  $|\Psi\rangle$  denoting the time-dependent state vector. We may split the hamiltonian,

$$\hat{H} \equiv \hat{H}_{\mathcal{P}} + \hat{H}_{\mathcal{PG}} + \hat{H}_{\mathcal{G}} , \quad (21)$$

where  $\hat{H}_{\mathcal{P}(\mathcal{G})}$  acts only on the partonic subsystem (gluonic environment) and  $\hat{H}_{\mathcal{PG}}$  describes the interaction connecting the two factorized subspaces introduced above, see eq. (2).

Until the end of this section we neglect the interaction term,  $\hat{H}_{\mathcal{PG}} \approx 0$ , and consider only an *exactly separable system* for simplicity. Furthermore, let us *assume* that a Schmidt basis of  $\mathcal{P}$  simultaneously diagonalizes  $\hat{H}_{\mathcal{P}}$ . This implies

$$[\hat{\rho}_{\mathcal{P}} , \hat{H}_{\mathcal{P}}] = 0 , \quad \text{and} \quad \partial_t \hat{\rho}_{\mathcal{P}} = 0 . \quad (22)$$

Thus, we may write  $\hat{H}_{\mathcal{P}} = \sum_p E_p |p\rangle\langle p|$ , where  $E_p$  are the associated energy eigenvalues. Then, the most general  $\hat{\rho}_{\mathcal{P}}$  allowed by eqs. (22) and orthonormality of the Schmidt states is of the form given by eq. (12). It includes the interesting case of a “pseudo-thermal ensemble”,

$$\hat{\rho}_{\mathcal{P}}(\beta_p) \equiv \frac{\exp\{-\sum_p \beta_p E_p |p\rangle\langle p|\}}{\text{Tr}_{\mathcal{P}} \exp\{-\sum_p \beta_p E_p |p\rangle\langle p|\}} = \sum_p \frac{e^{-\beta_p E_p}}{Z} |p\rangle\langle p| , \quad (23)$$

where  $Z(\beta_p) \equiv \sum_p \exp\{-\beta_p E_p\}$ ; for  $\beta_p \rightarrow \beta$  the usual density operator of a canonical ensemble results [23]. Of course, we find a non-zero entropy here by eqs. (18) or (19). We remark that eqs. (22) by themselves imply that the expansion coefficients (probabilities) in eqs. (12,13) are constant,  $\partial_t c_p = 0$ . However, from our assumption that  $\{|p\rangle\}$  diagonalizes  $\hat{H}_{\mathcal{P}}$ , we cannot further conclude  $[\hat{\rho}_{\mathcal{G}} , \hat{H}_{\mathcal{G}}] = 0$ . In general, depending on the initial state  $|\Psi_0\rangle$  in eq. (20), the Schmidt basis states  $|p_{\mathcal{G}}^n\rangle \in \mathcal{G}$  will show a complicated time-dependence.

Exact separability, naturally, cannot be expected to correspond to physically interesting situations in any realistic way: (A) Consider two decoupled field theories, one of which, the partonic subsystem, is additionally assumed to be essentially a free theory, cf. eqs. (22) above; properly (anti)symmetrized partonic multiparticle states built from single parton momentum eigenstates constitute a suitable Schmidt basis here; coherent states do as well, if one neglects complications due to their overcompleteness. - Note that a Schmidt basis is, of course, not restricted to single-particle

states. - Or, (B) consider a metal as a free electron gas in a background lattice, which in turn can be described by a free phonon gas in the harmonic limit [23].

At this point we anticipate that a non-vanishing *interaction*  $\hat{H}_{\mathcal{P}\mathcal{G}}$  will significantly influence the evolution of the subsystem and its environment. In Sec. 3 we present analytical results for a rather simple interacting complex system. It turns out that under favourable conditions, loosely speaking in terms of the previous example (A), the partonic Schmidt basis remains rather stable with respect to the evolution of the complex system and, especially, allows a parton to be represented by a slowly evolving gaussian wavepacket. Thus, it behaves essentially as a *classical particle* under the influence of the interaction with the gluonic environment.

This result provides part of the motivation for our attempt to understand entropy production in strong interactions at high energy as being related to the quantum decoherence of the relevant Schmidt states, which would be *stable* in an idealized situation. Such states are commonly referred to as “*pointer states*” [7, 21]. Typically a pointer in a *classical* measuring apparatus corresponds to a preferred quantum pointer basis being selected dynamically, such that the interactions in the complex system (observed subsystem + environment + measuring apparatus) preserve the stable classical character of the pointer. Its positions on a scale can be described by states, the quantum superpositions of which are dynamically suppressed (meaning decoherence). Thus, in distinction to the well-defined Schmidt states, pointer states refer to a dynamical situation. Up to now no general criteria exist which guarantee their existence. Sometimes they may be realized only in an approximate way in a given complex system [21], which not necessarily has to be separable into a classically behaving subsystem plus its (quantum correlated) environment. The purpose of Sec. 4 is to begin to study these questions for partons in a gluonic environment.

### 3. Entropy Production via Pointer States in a Non-relativistic Model

#### 3.1 Time-evolution of the Density Submatrix

In the following we study a complex system of a particle linearly coupled to oscillators as a simple example to illustrate the concepts of the previous Section 2. For a most relevant review of previous work on the Caldeira-Leggett model in the context of quantum Brownian motion we refer to Ref. [26], see in particular Part III. Variants of this model have been applied before to the quantum decoherence problem (“reduction of the wave packet”) [24] and to the problem of radiation damping for a non-relativistic electron coupled to the quantized electromagnetic field [27], where special attention has been paid to the initial condition of the complex system [28].

For our purposes the *Caldeira-Leggett hamiltonian* can be written in analogy to eq. (21) with

$$H_{\mathcal{P}} \equiv \frac{p^2}{2M} \ , \quad H_{\mathcal{G}} \equiv \sum_{n=1}^N \left\{ \frac{p_n^2}{2m_n} + \frac{1}{2} m_n \omega_n^2 x_n^2 \right\} \ , \quad (24)$$

describing the partonic subsystem (“parton”) by a single free non-relativistic particle

(moving in one dimension for simplicity) and the gluonic environment by a set of  $N$  harmonic oscillators, and with the linear coupling given by

$$H_{\mathcal{PG}} \equiv -x \sum_{n=1}^N c_n \left\{ x_n - \frac{c_n}{2m_n\omega_n^2} x \right\} . \quad (25)$$

Thus,

$$H = \frac{p^2}{2M} + \sum_{n=1}^N \left\{ \frac{p_n^2}{2m_n} + \frac{1}{2} m_n \omega_n^2 \left( x_n - \frac{c_n}{m_n \omega_n^2} x \right)^2 \right\} . \quad (26)$$

Choosing  $c_n \equiv m_n \omega_n^2$ , we see that the model becomes explicitly translationally invariant [26, 27, 28], which presently also motivates the introduction of the parton “potential renormalization” term  $\propto x^2$  in eq. (25); it can be visualized as a “*parton moving along with its gluonic springs attached*”. In this form the model still allows to describe a large variety of physically interesting systems by choosing an appropriate spectral density for the environment [26],

$$I(\omega) \equiv \sum_{n=1}^N \frac{c_n^2}{2m_n\omega_n} \delta(\omega - \omega_n) = \frac{1}{2} \sum_{n=1}^N m_n \omega_n^3 \delta(\omega - \omega_n) . \quad (27)$$

As is well known, the environment induces noise and dissipation in the subsystem, which are essentially determined by  $I(\omega)$  (and initial conditions). Assuming a quasi-continuous distribution of environmental oscillators, spectral densities  $I(\omega) \propto \omega^k$  (for sufficiently small  $\omega$ ) with  $k > 0$  have been widely studied before, e.g. [24, 26, 28]. They correspond to so-called sub-Ohmic ( $k < 1$ ), Ohmic ( $k = 1$ ), and supra-Ohmic ( $k > 1$ ) environments, respectively. - The electron-radiation field system involves a supra-Ohmic environment with  $k = 3$  [27].

Here we consider a general spectral density in terms of an arbitrary dimensionless function  $F$ ,

$$I(\omega) \equiv g \Omega^3 F(\Omega^{-1} \omega) \Theta(\Omega - \omega) , \quad (28)$$

where  $g$  is a dimensionless coupling constant and  $\Omega$  denotes a high-frequency cutoff. For later purposes we introduce the (zero temperature) “noise” and “dissipation” kernels [24, 26],  $\nu$  and  $\eta$ ,

$$\nu(s) \equiv \int_0^\infty d\omega I(\omega) \cos(\omega s) , \quad \eta(s) \equiv - \int_0^\infty d\omega I(\omega) \sin(\omega s) ; \quad (29)$$

see, for example, eqs. (40,41) below for their dynamical effects. In the *short-time limit*,  $\Omega s \ll 1$ , we obtain:

$$\nu(s) = g_\nu \Omega^4 [1 + \mathcal{O}(\Omega^2 s^2)] , \quad \eta(s) = -g_\eta \Omega^4 [\Omega s + \mathcal{O}(\Omega^3 s^3)] , \quad (30)$$

with  $g_\nu \equiv g \int_0^1 dx F(x)$  and  $g_\eta \equiv g \int_0^1 dx x F(x)$ . This limit may be completely irrelevant for macroscopic quantum systems. However, assuming  $\Omega$  to be vaguely related to a separation of non-perturbative strong interactions with the gluonic environment from a perturbative regime at a scale on the order of  $\Lambda_{QCD} \approx 200$  MeV and relevant

time scales considerably less than 1 fm, we expect that something analogous to the short-time limit will be of interest there.

Having specified our toy model as above, our aim now is to calculate the density matrix  $\rho_{\mathcal{P}}$  of the partonic subsystem, see eq.(6), and to study the consequences of its time evolution. To achieve this, the dependence on the gluonic environment (harmonic oscillator degrees of freedom) has to be integrated out. In the Caldeira-Leggett model this can be done exactly using the Feynman-Vernon influence functional technique [23, 26], since the coordinates (and momenta) appear at most quadratically in the environmental hamiltonian and in the interaction, cf. eqs. (24,25).

Assuming the initial condition that the total density matrix factorizes,

$$\rho(0) \equiv \rho_{\mathcal{P}}(0) \cdot \rho_{\mathcal{G}}(0) \ , \quad (31)$$

where  $\rho_{\mathcal{P}}(0)$  describes the initial state of the parton, which will be defined below, and  $\rho_{\mathcal{G}}(0)$  denotes the density matrix of the gluonic environment which is assumed to be in the ground state at  $t_0 = 0$ ; i.e., the environmental oscillators only perform zero-point motion (“*vacuum fluctuations*”) initially. Then, the parton density matrix at a later time  $t$  is calculated with the appropriate propagator,

$$\rho_{\mathcal{P}}(x_f, x'_f, t) = \int dx_i dx'_i J(x_f, x'_f, t; x_i, x'_i, 0) \rho_{\mathcal{P}}(x_i, x'_i, 0) \ , \quad (32)$$

which has the path integral representation [26],

$$J(x_f, x'_f, t; x_i, x'_i, 0) = \frac{1}{Z} \int \mathcal{D}q \mathcal{D}q' e^{i(S_{\mathcal{P}}[q] - S_{\mathcal{P}}[q'])} e^{-\Phi[q, q']} \ , \quad (33)$$

with boundary conditions  $q(0) \equiv x_i$ ,  $q(t) \equiv x_f$ , and for  $q'$  analogously. Here  $Z$  is a normalization factor, which will conveniently be fixed at the end of the calculation, in order to preserve the normalization of  $\rho_{\mathcal{P}}$  or, equivalently, of the parton wave function;  $S_{\mathcal{P}}$  denotes the free parton action corresponding to  $H_{\mathcal{P}}$ , eq. (24),

$$S_{\mathcal{P}}[q] \equiv \int_0^t ds \frac{1}{2} M \dot{q}^2 \ , \quad (34)$$

and  $\Phi$  is the Feynman-Vernon influence functional describing the influence of the environment on the subsystem. In the present case it is completely determined by the noise and dissipation kernels [26], eqs. (29),

$$\begin{aligned} \Phi[q, q'] &= \int_0^t ds \int_0^s du [q(s) - q'(s)] \{ \nu(s-u)[q(u) - q'(u)] + i\eta(s-u)[q(u) + q'(u)] \} \\ &\quad + \Phi_{loc}[q, q'] \ , \end{aligned} \quad (35)$$

where the “localizing” part of the influence functional,

$$\Phi_{loc}[q, q'] \equiv i \sum_{n=1}^N \frac{c_n^2}{2m_n \omega_n^2} \int_0^t ds [q^2(s) - q'^2(s)] \ , \quad (36)$$

comes from the term  $\propto x^2$  in eq. (25). The constant appearing in eq. (36) can be calculated,

$$\frac{1}{2}M\omega_0^2 \equiv \sum_{n=1}^N \frac{c_n^2}{2m_n\omega_n^2} = \frac{1}{2} \sum_{n=1}^N m_n\omega_n^2 = \int_0^\infty \frac{d\omega}{\omega} I(\omega) = g_0\Omega^3 , \quad (37)$$

where we used eqs. (27,28) at intermediate steps and  $g_0 \equiv g \int_0^1 dx x^{-1} F(x)$ . Equation (37) shows that  $I(\omega)$  has to vanish sufficiently fast in the infrared, in order to avoid a divergence here.

In any case, we observe that the resulting path integrals are gaussian and, hence, can be evaluated exactly. Introducing new coordinates (with a unit Jacobian),  $y \equiv q - q'$  and  $z \equiv \frac{1}{2}(q + q')$ , one obtains from eq. (33):

$$J(y_f, z_f, t; y_i, z_i, 0) = \frac{1}{Z} \int \mathcal{D}y \mathcal{D}z e^{i\Sigma[y, z]} , \quad (38)$$

with the boundary conditions  $y(0) \equiv y_i = x_i - x'_i$ ,  $z(0) \equiv z_i = \frac{1}{2}(x_i + x'_i)$ , and  $y(t) \equiv y_f = x_f - x'_f$ ,  $z(t) \equiv z_f = \frac{1}{2}(x_f + x'_f)$ , and where the relevant total action now is given by

$$\Sigma[y, z] \equiv \int_0^t ds \left\{ M\dot{y}\dot{z} - M\omega_0^2 yz + \int_0^s du y(s-u)[i\nu(s-u)y(u) - 2\eta(s-u)z(u)] \right\} . \quad (39)$$

Since  $\nu$  is an even function, the second to last term here may be rewritten such that both integrations range from 0 to  $t$ .

The calculation of the path integrals proceeds in the standard way [23, 26]. Since they are gaussian, the essential coordinate dependence of the propagator can be obtained from the total action, eq. (39), evaluated with the extremal (“classical”) paths.<sup>5</sup> They are described by the usual Euler-Lagrange equations. Separate variations w.r.t.  $y$  and  $z$ , respectively, yield the equations of motion,

$$M\ddot{z}(s) + M\omega_0^2 z + 2 \int_0^s du \eta(s-u)z(u) = i \int_0^t du \nu(s-u)y(u) , \quad (40)$$

$$M\ddot{y}(s) + M\omega_0^2 y - 2 \int_s^t du \eta(s-u)y(u) = 0 . \quad (41)$$

Using the transformation  $\tilde{y}(t-s) \equiv y(s)$ , one finds that eq. (41) written in terms of  $\tilde{y}(\tilde{s} \equiv t-s)$  formally coincides with the homogeneous part of eq. (40). Thus, starting at  $s = t$ , it describes a corresponding motion backwards in time, however, with different boundary conditions. Since the equations are linear, real and imaginary parts of the solution for  $z$  can be calculated separately ( $y$  is real by eq. (41) and the boundary conditions). Fortunately, it turns out that the imaginary part of  $z$  does not contribute to the minimal action [26, 28] and, therefore, is irrelevant at present.

Then, we only have to solve one equation with the structure of the homogeneous real part of eq. (40) with appropriate boundary conditions. Its general solution can

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<sup>5</sup>An overall time-dependent factor stemming from the remaining integrations over gaussian fluctuations around the classical paths is absorbed into the normalization factor  $Z$ , see eq. (33).

be easily found in the short-time limit, cf. eqs. (30), with the Laplace transform technique,

$$z(s) = \frac{1}{f_-^2 + f_+^2} \left\{ z(0)[f_-^2 \cosh(f_- s) + f_+^2 \cos(f_+ s)] + \dot{z}(0)[f_- \sinh(f_- s) + f_+ \sin(f_+ s)] \right\}, \quad (42)$$

where  $f_{\pm} \equiv \Omega \{ \pm g_0 \Omega / M + [(g_0 \Omega / M)^2 + 2g_{\eta} \Omega / M]^{1/2} \}^{1/2}$  and  $g_0$  as introduced in eq. (37). Employing the transformation mentioned after eq. (41) and inserting the correct boundary conditions from eq. (38), we obtain:

$$z(s) = z_i [g_- \cosh(s_-) + g_+ \cos(s_+)] + \{z_f - z_i [g_- \cosh(t_-) + g_+ \cos(t_+)]\} \frac{f_- \sinh(s_-) + f_+ \sin(s_+)}{f_- \sinh(t_-) + f_+ \sin(t_+)}, \quad (43)$$

$$y(s) = y_f [g_- \cosh(t_- - s_-) + g_+ \cos(t_+ - s_+)] + \{y_i - y_f [g_- \cosh(t_-) + g_+ \cos(t_+)]\} \frac{f_- \sinh(t_- - s_-) + f_+ \sin(t_+ - s_+)}{f_- \sinh(t_-) + f_+ \sin(t_+)}, \quad (44)$$

with  $s_{\pm} \equiv f_{\pm} s$ ,  $t_{\pm} \equiv f_{\pm} t$  and  $g_{\pm} \equiv f_{\pm}^2 (f_-^2 + f_+^2)^{-1}$ . Note that in the short-time *weak-coupling limit*  $0 \leq s_{\pm} \leq t_{\pm} \ll 1$ . In this limit the results could be further simplified.

However, in the short-time limit generally we have to use the full expressions for the classical trajectories from eqs. (43,44), when calculating the minimal action. As mentioned above, only the real part of  $z$  contributes to it. Thus, after a partial integration of the first term on the r.h.s. of eq. (39) and making use of the real part of eq. (40), the minimal action is given by

$$\begin{aligned} \Sigma_{min}(y_f, z_f, t; y_i, z_i, 0) &= M (y_f \dot{z}_f - y_i \dot{z}_i) + \frac{1}{2} i \int_0^t ds \int_0^t du \nu(s-u) y(s) y(u) \\ &= M (y_f \dot{z}_f - y_i \dot{z}_i) + \frac{1}{2} i g_{\nu} \Omega^4 \left[ \int_0^t ds y(s) \right]^2, \end{aligned} \quad (45)$$

using (30) in the last step and where  $\dot{z}_{i,f} \equiv \dot{z}(s=0, t)$ , as determined by eq. (43). Inserting eqs. (43,44), the final result is:

$$\Sigma_{min}(y_f, z_f, t; y_i, z_i, 0) = \Sigma_R + i \Sigma_I, \quad (46)$$

in terms of the real and imaginary parts of the minimal action,

$$\begin{aligned} \Sigma_R &= \frac{M}{f_- \sinh(t_-) + f_+ \sin(t_+)} \left\{ [y_f z_f + y_i z_i] [f_-^2 \cosh(t_-) + f_+^2 \cos(t_+)] \right. \\ &\quad - y_i z_f [f_-^2 + f_+^2] \\ &\quad - y_f z_i [g_- f_-^2 + g_+ f_+^2 + 2g_- f_+^2 \cosh(t_-) \cos(t_+) + (g_+ - g_-) f_+ f_- \sinh(t_-) \sin(t_+)] \left. \right\} \\ &\equiv a y_f z_f + b y_i z_i + c y_f z_i + d y_i z_f, \end{aligned} \quad (47)$$

$$\begin{aligned}
\Sigma_I &= \frac{\frac{1}{2}g_\nu\Omega^4}{[f_- \sinh(t_-) + f_+ \sin(t_+)]^2} \{y_i[\cosh(t_-) - \cos(t_+)] \\
&\quad + y_f[\frac{2g_-f_+^2}{f_-f_+} \sinh(t_-) \sin(t_+) + (g_+ - g_-)(1 - \cosh(t_-) \cos(t_+))]\}^2 \\
&\equiv Ay_i^2 + By_iy_f + Cy_f^2 ,
\end{aligned} \tag{48}$$

and where we introduced the abbreviations for the time-dependent coefficients for later convenience. We remark that for the non-interacting case, i.e.  $g_0 = g_\nu = g_\eta = 0$ , the imaginary part vanishes, whereas the real part reproduces the well-known  $1/t$ -term,

$$\Sigma_R^0 = \frac{M}{t} \{y_f z_f + y_i z_i - y_f z_i - y_i z_f\} , \tag{49}$$

which characterizes the free particle density matrix propagator. Collecting the above results, eqs. (38) and (46) - (48), the propagator  $J$  immediately becomes

$$J(y_f, z_f, t; y_i, z_i, 0) = \frac{1}{\tilde{Z}(t)} e^{i\Sigma_R - \Sigma_I} . \tag{50}$$

The normalization factor  $\tilde{Z}$  (cf. the last footnote) will be calculated shortly.

We consider, in particular, a normalized gaussian wave packet as the initial state of a parton with momentum  $p$ ,

$$\psi(x_i, 0) \equiv \langle x_i, 0 | p \rangle \equiv \alpha e^{-\frac{1}{2}x_i^2/\beta^2} e^{ipx_i} , \tag{51}$$

with a corresponding density matrix,

$$\rho_{\mathcal{P}}(x_i, x'_i, 0) = \langle x_i, 0 | p \rangle \langle p | x'_i, 0 \rangle = \alpha^2 e^{-\frac{1}{2}(x_i^2 + x_i'^2)/\beta^2} e^{ip(x_i - x'_i)} . \tag{52}$$

Then, normalization initially requires

$$1 = \text{Tr}_{\mathcal{P}} \hat{\rho}_{\mathcal{P}} = \int_{-\infty}^{\infty} dx \rho_{\mathcal{P}}(x, x, 0) = \pi^{1/2} \alpha^2 \beta , \tag{53}$$

which fixes  $\alpha = \pi^{-1/4} \beta^{-1/2}$ . The normalization, however, has to be preserved through the time evolution of the system. Therefore, we proceed to calculate the time-dependent parton density matrix. It follows from eq. (32), using eqs. (47,48,50,52) and the coordinate transformation from eq. (38) above, after doing the two gaussian integrals,

$$\rho_{\mathcal{P}}(y_f, z_f, t) = \frac{\pi^{1/2}}{\tilde{Z}(t)\xi} e^{\left(\frac{1}{2}D(y_f, z_f, t)/\xi\right)^2 + iay_f z_f - \left(C + \frac{1}{4}\beta^2 c^2\right) y_f^2} , \tag{54}$$

with

$$D(y_f, z_f, t) \equiv idz_f + ip - (B + \frac{1}{2}\beta^2 bc)y_f , \quad \xi \equiv (A + \frac{1}{4}\beta^{-2} + \frac{1}{4}\beta^2 b^2)^{1/2} . \tag{55}$$

We remark that

$$|\psi(x, t)|^2 = \rho_{\mathcal{P}}(x, x, t) = \rho_{\mathcal{P}}(y_f = 0, z_f = x, t) = \frac{\pi^{1/2}}{\tilde{Z}(t)\xi} e^{-\frac{1}{4}(dx + p)^2/\xi^2} , \quad (56)$$

which, when integrated over  $x$ , fixes the normalization of the propagator. The result is:

$$\tilde{Z}(t) = 2\pi|d|^{-1} , \quad (57)$$

which, of course, has to come out independently of the chosen initial state.

Several interesting limits of the above results, which are generally valid in the short-time approximation, might be studied. It is instructive to evaluate the non-interacting case first of all. Inserting the time-dependent coefficients from eqs. (47,48), we obtain from eqs. (56,57) in the limit  $g \rightarrow 0$ :

$$\rho_{\mathcal{P}}^0(y_f = 0, z_f = x, t) = \pi^{-1/2} w_0^{-1}(t) e^{-(x - v_0 t)^2/w_0^2(t)} , \quad (58)$$

$$w_0(t) \equiv (\beta^2 + \beta^{-2} M^{-2} t^2)^{1/2} , \quad v_0 \equiv \frac{p}{M} , \quad (59)$$

which shows the well-known spreading of the wave packet due to the time-dependent width  $w_0$  and its motion with velocity  $v_0$  according to the classical law. By comparison with eqs. (56,57) we find the corresponding width and velocity for the interacting system,

$$w(t) \equiv 2\xi|d|^{-1} , \quad v(t) \equiv -pd^{-1}t^{-1} . \quad (60)$$

Thus, using eqs. (54) - (57) and (60), the full density matrix becomes

$$\begin{aligned} \rho_{\mathcal{P}}(y_f, z_f, t) &= \pi^{-1/2} w^{-1}(t) e^{-(z_f - v(t)t)^2/w^2(t)} \\ &\cdot e^{-y_f^2 \left( C + \frac{1}{4}\beta^2 c^2 - d^{-2} [B + \frac{1}{2}\beta^2 bc]^2/w^2(t) \right)} \\ &\cdot e^{iy_f \left( az_f - 2d^{-1} [B + \frac{1}{2}\beta^2 bc] (z_f - v(t)t)/w^2(t) \right)} , \end{aligned} \quad (61)$$

which for  $g \rightarrow 0$  reduces to

$$\begin{aligned} \rho_{\mathcal{P}}^0(y_f, z_f, t) &= \rho_{\mathcal{P}}^0(y_f = 0, z_f, t) \cdot e^{-\frac{1}{4}y_f^2 \beta^2 M^2 t^{-2} [1 - \beta^2/w_0^2(t)]} \\ &\cdot e^{iy_f M t^{-1} (z_f [1 - \beta^2/w_0^2(t)] + v_0 t \beta^2/w_0^2(t))} , \end{aligned} \quad (62)$$

cf. eqs. (58,59).

### 3.2 Quantum Decoherence in the Short-time Strong-coupling Limit

Next, we turn to a discussion of the *short-time strong-coupling limit*, which we define through the following conditions:

1.  $\Omega t \ll 1$  (“short time”) ;
  2.  $g_0 \gg 1$  ,  $g_0/g_{\nu,\eta} \gg 1$  (“strong coupling”) ;
  3.  $\Omega/M \gg 1$  .
- (63)



Note that the second of conditions 2. implies that the function  $F$  specifying the spectral density  $I$ , eq. (28), has to be strongly infrared dominated by the definitions of  $g_{\nu,\eta}$  and  $g_0$  following eqs. (30) and (37), respectively. Condition 3. is added mainly for convenience. From conditions 1. - 3. we obtain

$$f_+^2 \approx 2g_0(\Omega/M)\Omega^2, \quad f_+ \gg f_- , \quad g_+ \approx 1, \quad g_- \ll 1, \quad (64)$$

where we used the definitions of  $f_{\pm}$  and  $g_{\pm}$  following eqs. (42) and (44), respectively. To be definite, we will be interested particularly in times  $t$  such that  $f_+t \leq O(1)$ . Some approximate relations follow here for the time-dependent coefficients defined in eqs. (47,48),

$$a = b \approx Mf_+ \cot(f_+t), \quad c \approx d \approx -Mf_+/\sin(f_+t),$$

$$A \approx C \approx \frac{1}{2}B \approx \frac{1}{2}g_{\nu}\Omega^4[1 - \cos(f_+t)]^2/[f_+^2 \sin^2(f_+t)], \quad (65)$$

which will be useful henceforth. Note that all of eqs. (65) approach the correct limit for  $g \rightarrow 0$  or  $t \rightarrow 0$ . Evaluating eqs. (60) in the short-time strong-coupling limit, using (65), one finds

$$v(t) \approx \frac{p}{M} \frac{\sin(f_+t)}{f_+t} = v_0 \frac{\sin(f_+t)}{f_+t}, \quad (66)$$

$$w^2(t) \approx \beta^2[\cos^2(f_+t) + \frac{g_{\nu}}{2g_0^2(\beta\Omega)^2}(1 - \cos(f_+t))^2] + \beta^{-2}M^{-2}f_+^{-2}\sin^2(f_+t). \quad (67)$$

Generally, the results show several remarkable features, if one compares the expression for the density matrix obtained from eq. (61) in the present limit with the non-interacting case given in eq. (62).

To begin with, all time-dependent functions entering eqs. (60,61) through the coefficients evaluated in eqs. (65) are governed by a single *dynamical time scale*  $f_+^{-1}$ , cf. (64). From previous experience we expect that their precise form is sensitive to the initial condition [24, 26, 28], particularly the simple factorization in eq. (31). The fact that in the present limit trigonometric functions dominate over the hyperbolic ones in eqs. (47,48), which usually would rather determine the long-time behaviour of the propagator etc., also points towards this conclusion. We remark that for  $t \rightarrow 0$  eq. (61) yields  $\rho_{\mathcal{P}} \rightarrow \rho_{\mathcal{P}}^0$ , eq. (62), in agreement with the factorized initial condition. Furthermore, there is obviously and quite naturally a dependence on the initial state of the parton parametrized by the width  $\beta$  and momentum  $p$ . Note, however, had we chosen, for example, a gaussian superposition in momenta instead of eq. (51),

$$\tilde{\psi}(x_i, 0) \equiv \tilde{\alpha} e^{-\frac{1}{2}x_i^2/\beta^2} \int_{-\infty}^{\infty} dp' e^{ip'x_i} e^{-(p' - p)^2/P^2}, \quad (68)$$

this would simply replace  $\xi^2 \rightarrow \xi^2 + \frac{1}{4}P^2$ , cf. (55), and leave the density matrix the same otherwise, eq. (54), apart from a constant overall factor. We will consider another, more interesting parton initial state below.

Turning to the diagonal elements of the density matrix,  $\rho_{\mathcal{P}}(0, z_f, t)$  from eq. (56) and (61) in particular, we observe that they are completely determined by the velocity and width,  $v$  and  $w$  as obtained in eqs. (66) and (67), respectively:

(i) The free velocity  $v_0$  is modified by a slowly decreasing function of time (scaled by  $f_+$ ), which shows an influence of the gluonic environment oscillators on the parton as an effective *friction* force.

(ii) Compared to the spreading of the wave packet of a free particle, see  $w_0$  in (59), the width has become a very slowly evolving function of time in the region of interest to us,  $f_+ t \leq O(1)$ , and can actually be smaller than the initial width  $w(0) = w_0(0) = \beta$  for a suitable choice of the parameters (*localization*); note that the term  $\propto g_\nu$ , which is due to the noise kernel, cf. (29), presents an increasing contribution to  $w$  which can be neglected, if the initial width is not excessively too small.

In fig. 1 we illustrate the above results by showing the unique *deceleration function*  $v/v_0$ , see eq. (66), as a function of the dimensionless variable  $t_+ \equiv f_+ t = \Omega t (2g_0 \Omega / M)^{1/2}$ . Furthermore, we show the relative *change of the width* of the parton wave packet, cf. eq. (67), as a function of the same variable:

$$\frac{w(t_+) - \beta}{\beta} = \left\{ \cos^2(t_+) + \alpha_{<} [1 - \cos(t_+)]^2 + \alpha \sin^2(t_+) \right\}^{1/2} - 1 \quad , \quad (69)$$

with the parameters given by

$$\alpha \equiv \frac{1}{2g_0} \frac{\Omega}{M} (\beta \Omega)^{-4} \quad , \quad \alpha_{<} \equiv \alpha \cdot \frac{g_\nu}{g_0} \frac{M}{\Omega} (\beta \Omega)^2 \quad . \quad (70)$$

Note that the expression corresponding to eq. (69) for a *free* particle ( $g = 0$ ) can be written similarly,

$$\frac{w_0(t_+) - \beta}{\beta} = \left\{ 1 + \alpha t_+^2 \right\}^{1/2} - 1 \quad , \quad (71)$$

which is also shown in one case in fig. 1 for comparison.

By modifying the spectral function of our toy model, eq. (28), one could optimize the above results in the sense that (i) the environmental friction is minimized and simultaneously (ii) the stability of the parton wave packet is maximized. It is, however, a general property of linear coupling models à la Caldeira-Leggett that friction and localization go together to some extent [26]. We remind ourselves that for supra-Ohmic environments with spectral exponent  $k > 2$ , as discussed after eq. (27), one even finds an effectively free quantum particle with renormalized properties in the long-time limit. Interestingly, this applies for a non-relativistic electron coupled to the quantized electromagnetic field, where  $k = 3$  [27]. Thus, as could be expected, our parton toy model specified by eqs. (28) - (30) and (63), in particular, behaves very different from the electron-radiation field system.

At this point we want to pause for a moment and turn our attention back to the discussion of partonic Schmidt and pointer states in Sec. 2 before. After all, the toy model in this section is meant to illustrate those concepts and to provide an explicit example, how they may be realized in a physical system, which is vaguely oriented towards QCD partons. It should have become clear that the single-parton wave packet,

eq. (51), trivially constitutes a *Schmidt basis* for the single-parton (plus gluonic environment) system under consideration, cf. eqs. (11) - (13). However, far less trivial seems the fact that this Schmidt basis also is dynamically rather stable in the sense of points (i) and (ii) above. Thus, a parton here behaves in a good approximation as a *classical particle* moving in a dissipative environment. Hence, we have shown that the Schmidt basis defined by the chosen initial wave packet state also presents an approximate realization of a *pointer state* basis. To complete this demonstration, we have to show that these pointer states have appropriate decoherence properties as discussed after eq. (16).

Thus, instead of considering the off-diagonal density matrix elements from eq. (61) (for  $y_f \neq 0$ ) in detail,<sup>6</sup> we now consider a different parton initial state. Our aim is to show that

(iii) the gluonic environment enforces the evolution of a *pure state* consisting of a *coherent superposition of pointer states* into a *mixed state* consisting of a *decoherent sum* related to those states; i.e., the interference terms of the initial density matrix become dynamically suppressed as time goes on.

This is the **origin of entropy production** in the present model, as we shall see.

Instead of the parton initial state defined in eq. (51), we want to study the coherent superposition of two such wave packets with momenta  $+\tilde{p}$  and  $-\tilde{p}$ :

$$\tilde{\psi}(x_i, 0) \equiv N^{-1/2}[\psi_{+\tilde{p}}(x_i, 0) + \psi_{-\tilde{p}}(x_i, 0)] \quad , \quad N \equiv 2[1 + e^{-(\tilde{p}\beta)^2}] \quad , \quad (72)$$

where the index  $\pm\tilde{p}$  is introduced to indicate the respective momenta and  $N$  is the necessary additional normalization constant, see eqs. (51) - (53) for comparison. Equation (72) yields the corresponding initial density matrix,

$$\begin{aligned} \tilde{\rho}_{\mathcal{P}}(x_i, x'_i, 0) &= N^{-1} \tilde{\psi}(x_i, 0) \tilde{\psi}^*(x'_i, 0) \\ &= N^{-1} [\rho_{\mathcal{P}}(x_i, x'_i, 0)_{+\tilde{p}} + \rho_{\mathcal{P}}(x_i, x'_i, 0)_{-\tilde{p}} + \rho_{int}(x_i, x'_i, 0)] \quad , \end{aligned} \quad (73)$$

with the pure state density matrix elements  $\rho_{\mathcal{P}}(x_i, x'_i, 0)_{\pm\tilde{p}}$  as in eq. (52) and the interference term

$$\rho_{int}(x_i, x'_i, 0) \equiv \alpha^2 e^{-\frac{1}{2}(x_i^2 + x_i'^2)/\beta^2} [e^{+i\tilde{p}(x_i + x'_i)} + e^{-i\tilde{p}(x_i + x'_i)}] \quad . \quad (74)$$

Then, the time evolution of  $\tilde{\rho}_{\mathcal{P}}$ , which follows the linear law of eq. (32), can be evaluated term by term. The result for  $\rho_{\mathcal{P} \pm\tilde{p}}$  is given by eq. (61) with the appropriate momentum inserted, whereas the *interference term* at a later time  $t$  is calculated to be

$$\rho_{int}(y_f, z_f, t) = \sum_{\pm} \rho_{\mathcal{P}}(y_f, z_f, t)|_{v \rightarrow 0, \quad cy_f \rightarrow c(y_f \pm 2\tilde{p}/c)} \quad . \quad (75)$$

Here  $\rho_{\mathcal{P}}$  is given by eq. (61) as well, however, with the indicated substitutions, which can easily be understood. Since presently  $v \rightarrow 0$ , i.e.  $p = 0$ , there is no motion

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<sup>6</sup> Generally, one finds with the help of eqs. (65) an *increased* gaussian (in  $y_f$ ) fall-off as compared to the non-interacting case, eq. (62), with time-dependences governed by the scale  $f_+^{-1}$  discussed above, and corresponding modifications of the phase factor.

of the gaussian in the center coordinate  $z_f$ , compare eq. (61). On the other hand, the substitutions  $cy_f \rightarrow c(y_f \pm 2\tilde{p}/c)$  corresponding to the respective parts of the interference term, eq. (74), imply that parts of the resulting gaussian in the relative coordinate  $y_f$ , see eq. (61), “move away” with a velocity proportional to the relative momentum  $2\tilde{p}$  of the superposed initial wave packets. There is also a similar effect on the phase factor. Because of the complexity of the r.h.s. of eq. (75), it is not illuminating to examine the behaviour of individual density matrix elements obtained from eq. (73) at a later time  $t$  with the help of eqs. (61) and (75).

Therefore, we proceed to calculate directly the entropy for the various interesting states. In particular, we employ the *linear entropy* defined in eq. (19) for two reasons: a)  $S_{\mathcal{P}}^{lin}$  has a simple bound from above, which eventually gives a good qualitative impression of the saturation of entropy production; b) practically, for any somewhat complicated non-diagonal density matrix, it seems impossible to calculate the standard entropy  $S_{\mathcal{P}}$  according to eq. (18) without performing a more or less difficult diagonalization first of all.

Let us begin by calculating the linear entropy  $S_1^{lin}$  for the simple pointer state (51) with its density matrix (61) using eq. (19):

$$\begin{aligned} S_1^{lin} &= 1 - \text{Tr}_{\mathcal{P}} \hat{\rho}_{\mathcal{P}}^2 = 1 - \int_{-\infty}^{\infty} dy_f \int_{-\infty}^{\infty} dz_f \rho_{\mathcal{P}}(y_f, z_f, t) \rho_{\mathcal{P}}(-y_f, z_f, t) \\ &= 1 - \frac{1}{2} c_1^{-1/2} w^{-1} , \end{aligned} \quad (76)$$

$$c_1 \equiv C + \frac{1}{4} \beta^2 c^2 - d^{-2} w^{-2} [B + \frac{1}{2} \beta^2 b c]^2 , \quad (77)$$

independently of the parton momentum  $p$ . In the non-interacting limit ( $g \rightarrow 0$ ), where the previously defined time-dependent functions reduce to  $A = B = C = 0$ ,  $a = b = -c = -d = M/t$ , and  $w \rightarrow w_0$ ,  $v \rightarrow v_0$ , eqs. (59), the r.h.s. of eq. (76) correctly goes to zero for *all times*. This is in agreement with eq. (19), since here the initial pure state remains a pure state, which has zero entropy. However, our proposed pointer state (51), is not perfectly stable. Consequently, we expect some amount of entropy to be generated due to the interaction with the gluonic environment. Indeed, in the short-time strong-coupling limit (63) using eqs. (65), one obtains from eq. (76)

$$S_1^{lin}(t) \approx \frac{1}{4} \frac{g_{\nu}}{g_0} \left[ \frac{1}{2g_0(\beta\Omega)^2} [1 - \cos(f_+t)]^2 + \frac{(\beta\Omega)^2}{\Omega/M} \sin^2(f_+t) \right] , \quad (78)$$

which is very small for suitable  $\beta\Omega = \mathcal{O}(1)$  and according to the assumptions (63). In this sense the pointer state (51) conforms with the so-called *predictability sieve* [25], which characterizes pointer states as producing the least possible entropy increase. Furthermore, note that a perfect pointer state would have to diagonalize the relevant part of the total hamiltonian,  $H_{\mathcal{P}} + H_{\mathcal{P}\mathcal{G}}$ , as defined in eqs. (24,25). We did not seriously try to improve the wave function (51) to optimize its pointer state character. However, apart from the points (i) and (ii) discussed above, we will find additional support for having made a reasonable guess by its decoherence properties.

For this purpose we calculate the linear entropy  $S_2^{lin}$  for the coherent superposition of two pointer states, eq. (72), with its density matrix  $\tilde{\rho}$  at a finite time  $t$  following from eqs. (61) and (73) - (75) as described after eq. (74). The result of a lengthy but

straightforward calculation, which involves a number of double gaussian integrations as in eq. (76), can be represented as follows:

$$S_2^{lin} = 1 - \text{Tr}_{\mathcal{P}} \tilde{\rho}_{\mathcal{P}}^2 = 1 - N^{-2} \text{Tr}_{\mathcal{P}} \{ \hat{\rho}_{\mathcal{P}+\tilde{p}} + \hat{\rho}_{\mathcal{P}-\tilde{p}} + \hat{\rho}_{int} \}^2 = \quad (79)$$

$$1 - N^{-2} c_1^{-1/2} w^{-1} \left\{ [2 \cdot \frac{1}{2} + e^{-2(\tilde{p}/dw)^2 - \frac{1}{2}c_6^2/c_1}] + [e^{2c_3 + \frac{1}{2}c_2^2/c_1} + e^{-2(\tilde{p}\beta)^2}] \right.$$

$$\left. + [4 \cos(\frac{1}{2}c_5\tilde{p}/d + \frac{1}{4}c_2c_6/c_1) e^{-\frac{1}{2}(\tilde{p}/dw)^2} + (c_2^2 + 8c_3c_1 - c_6^2 - w^2c_5^2c_1)/8c_1] \right\} ,$$

where we grouped terms into square brackets which come from  $(\hat{\rho}_{\mathcal{P}+\tilde{p}} + \hat{\rho}_{\mathcal{P}-\tilde{p}})^2$ ,  $\hat{\rho}_{int}^2$ , and the cross terms involving  $\rho_{int}$  at last, respectively;  $N$  was defined in eq. (72),  $c_1$  in eq. (77), and the other abbreviations are

$$c_2 \equiv -c\tilde{p}\beta^2[1 - \beta^2b^2d^{-2}w^{-2} - 2Bbc^{-1}d^{-2}w^{-2}] , \quad c_3 \equiv -\tilde{p}^2\beta^2[1 - \beta^2b^2d^{-2}w^{-2}] ,$$

$$c_5 \equiv -2\tilde{p}\beta^2bd^{-1}w^{-2} , \quad c_6 \equiv -2\tilde{p}d^{-2}w^{-2}[B + \frac{1}{2}\beta^2bc] . \quad (80)$$

We should, of course, check again that in the non-interacting limit there is no entropy produced, since the pure state  $\tilde{\psi}$ , eq. (72), which corresponds to  $\tilde{\rho}$ , remains a pure state. In fact, it is instructive to see, how this result comes about,

$$S_{2g=0}^{lin} = 1 - 2N^{-2} \left\{ [2 \cdot \frac{1}{2} + e^{-2(\tilde{p}\beta)^2}] + [1 + e^{-2(\tilde{p}\beta)^2}] + [4 e^{-(\tilde{p}\beta)^2}] \right\} = 0 , \quad (81)$$

where the terms in square brackets are in one-to-one correspondence with those in eq. (79). We observe that all terms are exponentially small except for three contributions: two identical terms in the first bracket, which can be interpreted as a decoherent sum for the two superposed pointer states, cf. eqs. (76,79); however, there is also a large term originating from the square of the interference term,  $\hat{\rho}_{int}^2$  in eq. (79), which appears in the second bracket. Obviously, if there is any considerable entropy production to occur in the interacting case, then particularly this contribution of the interference term has to be significantly reduced, which essentially amounts to the *decoherence effect* that we are looking for.

Using the full result, eqs. (79,77), evaluated in the short-time strong-coupling limit and expanding consistently to lowest non-trivial order in quadratically small quantities, i.e. terms which are proportional to  $g_\nu/g_0^2$  or  $g_\nu/(g_0\Omega/M)$ , we find:

$$S_2^{lin}(t) \approx S_1^{lin}(t) - 2N^{-2} \left\{ \sigma_1 e^{-2(\tilde{p}\beta)^2} - \sigma_2 + 4\sigma_3 e^{-(\tilde{p}\beta)^2} \right\} , \quad (82)$$

where we also used eqs. (76) - (78) and assumed  $\beta\Omega = O(1)$  again; furthermore, the terms  $\propto \sigma_{1,2,3}$  correspond to the square brackets in eqs. (79) or (81) in an obvious way, with

$$\sigma_1 \equiv (\tilde{p}\beta)^2 \frac{2g_\nu(\beta\Omega)^2}{g_0\Omega/M} \sin^2(f_+t) ,$$

$$\sigma_2 \equiv (\tilde{p}\beta)^2 \frac{g_\nu}{g_0^2(\beta\Omega)^2} [1 - \cos(f_+t)]^2 ,$$

$$\sigma_3 \equiv (\sigma_1 - \sigma_2)/4 . \quad (83)$$

Several remarks are in order here. Firstly, note that all  $\sigma$ 's are  $\propto \tilde{p}^2$ . Thus,  $\tilde{p} \rightarrow 0$  correctly implies  $S_2^{lin}(t) \rightarrow S_1^{lin}(t)$ , since in this limit we recover the simple (imperfect) pointer state (51) with  $p = 0$  and its associated entropy, see eqs. (76) - (78). Secondly, for moderately large relative momentum between the two coherently superposed pointer states in eq. (72) as compared to their spatial width, i.e.  $\tilde{p} > \beta^{-1}$ , we obtain from eqs. (82,83) the final result:

$$S_2^{lin}(t) \approx \frac{1}{2}\sigma_2 . \quad (84)$$

Thus, we conclude that an appreciable amount of linear *entropy* can be produced here through the mechanism of *environment-induced decoherence of coherently superposed (approximate) pointer states*. In particular, note that the dominant contribution  $\propto \sigma_2$  comes from the decay of the interference term, which we anticipated in the discussion following eq. (81). The rate is determined by the dynamical time scale  $f_+^{-1}$ , cf. (64), which also governs the friction and localization effects mentioned in points (i) and (ii) above.

It is worth noting that in the limit of very large relative momentum,  $\tilde{p}\beta \rightarrow \infty$ , we find from eq. (79) in the short-time strong-coupling limit instead of (84):

$$S_2^{lin}(t) \approx 1 - \frac{1}{2}[1 - S_1^{lin}(t)][2 \cdot \frac{1}{2}] = \frac{1}{2}[1 + S_1^{lin}(t)] . \quad (85)$$

This result corresponds simply to the expected decoherent superposition of two equally probable states, cf. eq. (19), which would yield  $S_p^{lin} = 1 - \{(\frac{1}{2})^2 + (\frac{1}{2})^2\}$ . Again there is a small correction due to the imperfection of our pointer states as discussed above.

Finally, we remark that for the demonstration of entropy production or environment-induced decoherence we did not necessarily choose the *optimal* superposition of approximate pointer states in the sense of the largest or fastest effect. Our above result, eqs. (82) - (84), shows that the entropy production is larger for superposed states which are further separated in momentum, i.e. states the wave packets of which have consequently moved further apart in space after a given amount of time has passed. From this observation and the fact that the interaction between partonic subsystem and gluonic environment in our toy model, see eqs. (25,26), acts on the respective coordinates rather than the momenta one may be tempted to speculate that the superposition of initially spatially (rather than in momentum) separated pointer states would decohere even faster and, thus, produce entropy at a higher rate. Indeed, in the different context of Ref. [24], such an effect of a *preferred observable* (coordinate vs. momentum) has been found.

These remarks complete the demonstration of dynamically enforced decoherence of coherent superpositions of pointer states, point (iii) above, and its relation to entropy production for our toy model.

More general superpositions of states can, of course, be studied similarly. In particular, a general parton initial state could be decomposed into coherent sums over pairs of localized gaussian wave packets (see also the discussion in Ref. [24]), for which the present considerations apply again.

The extension to a *multi-parton system* is straightforward only as long as we can neglect perturbative interactions among themselves<sup>7</sup> and interactions mediated through the self-coupling of the gluonic environment. Otherwise, provided that a Feynman-Vernon type influence functional can still be calculated (cf. Sec. 3.1), one has to face at least a generally quite difficult mechanical  $n$ -body problem, i.e. a large set of coupled equations replacing (40,41). This seems to prohibit a stepwise development of our toy model into a more realistic phenomenological model. Therefore, we propose a fully relativistic quantum field theory model instead in the following section, which is motivated by the non-relativistic model studied here. It might mimic some features of QCD and the parton model.

Generally, we expect to encounter the set of characteristic formal problems posed by the requirements of Lorentz and local gauge covariance as well as by the occurrence of the usual divergences of quantum field theory. A more subtle difference, which is due to going over from the non-relativistic Schrödinger equation to proper wave equations describing the basic fields, will mostly be hidden in our functional Schrödinger picture approach.

## 4. Decoherence and Entropy Production in a Scalar “Parton” Field Theory

### 4.1 Model of Momentum Space Mode Separation

Before attempting to generalize our previous considerations to QCD gauge theory, we want to take an intermediate step here by studying a much simpler model of two coupled real scalar fields,  $\phi_1$  and  $\phi_2$ , representing partons and “unobservable” gluonic environment modes, respectively. It is defined by the classical action

$$S \equiv \int d^4x \{L_1 + L_{12} + L_2\} \ , \quad (86)$$

with ( $g_{\mu\nu} \equiv \text{diag}[1, -1, -1, -1]$ )

$$L_j \equiv \frac{1}{2}(\partial\phi_j)^2 - v_j(\phi_j) \ , \quad j = 1, 2 \ ; \quad L_{12} \equiv -V(\phi_1, \phi_2) \ , \quad (87)$$

i.e., we consider an *attractive interaction* of the two scalar fields, in particular. Clearly, the meaning of the model depends on how we define the separation of partons and environment modes.

To begin with, let us assume that there was only *one* scalar quantum field  $\varphi$  representing the *total* system. Then, similar to the background field method, we may split it,  $\varphi \equiv \phi_1 + \phi_2$ , according to the following considerations.

The essential feature of QCD to be incorporated here phenomenologically is the *running coupling constant*. “Soft” interactions with small four-momentum transfer (squared,  $|Q|^2$ ) have a strong effective coupling and, generally, cannot be described in perturbation theory. Conversely, “hard” processes with large  $|Q|^2$  are reliably

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<sup>7</sup>This is a question of the relative sizes of a non-perturbative time scale corresponding to  $f_+^{-1}$  here and a perturbative “rescattering” time, which has to be taken up in the QCD context

accounted for by perturbation theory and constitute the *only* basis for the comparison of QCD theory with experiment [29]. Field modes which are sufficiently close to constant field configurations and *would* interact only among themselves form a sector of the total Hilbert space which *would* be completely unobservable. They can neither hadronize nor can any large- $|Q|^2$  process occur *within* this sector due to the restriction of the four-momenta. Roughly, we assume  $k_0^2 < m_\pi^2$  and  $\vec{k}^2 < \Lambda_{QCD}^2$ . Of course, there is no sharp boundary of this sort in QCD. In any case, this defines the *gluonic environment* described by the field  $\phi_2$  and  $L_2$  in eqs. (86,87) and the region II in fig. 2. (We don't distinguish soft quark modes presently.) The complement of this part of the total Hilbert space, region I in fig. 2, is described by  $\phi_1$  and  $L_1$  and constitutes the *parton subsystem*, which is almost directly observable by hadronization or via large- $|Q|^2$  processes. Finally, we describe the interactions between partons and environment by  $L_{12}$  in eqs. (86,87).

Several qualifying remarks are in order here:

1. The Momentum Space Mode Separation (“MSMS”) as introduced above is *not* Lorentz invariant. Therefore, it only makes sense in a certain frame. It should be the one in which we formulate the initial conditions for the time-evolution problem of our field theory (cf. Sec. 4.4).
2. In a non-abelian gauge theory MSMS is *not* gauge invariant. Therefore, it only makes sense, if the gauge is completely fixed; particularly, if the theory is described in terms of physical degrees of freedom.
3. At present  $\phi_1$  and  $\phi_2$  are *quantum* fields, whereas usually in the background field method the background consists of a classical field, which does not propagate in quantum loops, whereas the quantum field only lives on internal lines in Feynman diagrams by construction. Obviously, a separation of field modes according to some dynamical criterion (e.g. MSMS) requires a non-trivial modification of the standard background (gauge) field method [20].
4. There is an analogy to the non-relativistic model of Sec. 3; the interaction  $L_{12}$  may be depicted as the “*partons dragging along the unobservable background field modes*” or vice versa. A close analogy would be a non-local interaction involving a form factor,

$$\int d^4x \tilde{L}_{12} \equiv \int d^4x d^4x' [\phi_1(x) - \phi_2(x)] \omega^2(x - x') [\phi_1(x') - \phi_2(x')] ,$$

which is, however, local in momentum space and, thus, would rather directly generalize the interaction in eq. (26). Unfortunately, such an effective interaction presents difficulties, when attempting the usual equal-time quantization of the model. Therefore, it is also not obvious how to write down a functional Schrödinger equation for it, cf. eq. (89) below, and we will not consider it any further.

In order to specify our model, we consider the potentials in eqs. (87) in more detail according to MSMS. Renormalizability and stability require the potentials to be fourth order polynomials. Then, a minimal version of the model may be defined by:

$$v_1(\phi_1) \equiv \frac{1}{4!} \lambda_1 \phi_1^4 \quad , \quad v_2(\phi_2) \equiv -\frac{1}{2} \mu^2 \phi_2^2 + \frac{1}{4!} \lambda_2 \phi_2^4 \quad ,$$



$$V(\phi_1, \phi_2) \equiv \lambda_{12} \phi_1^2 \phi_2^2, \quad (88)$$

with the restriction  $0 < \lambda_1 < \lambda_{12} < \lambda_2$ , which is supposed to mimic in a very schematic way the  $|Q|^2$ -dependence of the strong coupling. In  $v_2(\phi_2)$  we included a negative “mass” term to allow for a condensation of environment modes (generating different masses for  $\phi_1$  and  $\phi_2$ ). The  $\phi_2$ -potential is fully consistent with MSMS.

However, in the mutual coupling we encounter the problem that it admits, for example, the annihilation of two space-like partons into two environment gluons with arbitrarily small  $|Q|^2$ . This contradicts a clear-cut MSMS, such that the respective size of couplings in (88) corresponds to the  $|Q|^2$ -dependent situation in QCD. The same effect would arise in other couplings, which we presently did not include for simplicity, and cannot be avoided in such a crude model. Similarly, the  $\phi_1^4$ -interaction admits some low- $|Q|^2$  processes, which it should not. Therefore, only on the average (in multiple scattering) may we hope to mimic a bit of the QCD case. This will be studied in the sequel to the present work. There we may want to include also  $\phi_1\phi_2^2$ - and  $\phi_1^2\phi_2$ -couplings. They correspond to decay/fusion and soft scattering processes, which are important in parton cascades [11, 12, 18]. In the time-dependent Hartree-Fock approximation considered below the tree level Feynman diagrams for the two-point functions corresponding to the latter cubic couplings would simply carry one classical field  $\langle\phi_{1,2}\rangle$  on one leg, whereas  $V(\phi_1, \phi_2)$  as in (88) describes effective mass insertions  $\langle\phi_i^2\rangle_{i=1,2}$  and something like a formfactor  $\langle\phi_1\phi_2\rangle$ , which we discussed above and which we consider most interesting to start with.

Generally, the self-interactions  $v_j(\phi_j)$  and the mutual coupling  $V(\phi_1, \phi_2)$  necessitate a *renormalization* of our model. It is well known, how to carry out the standard program for scalar fields. Furthermore, it has been extended to the study of initial-value problems recently [30], which would be relevant in the following. However, since it does not illuminate our main topic here, we simply assume for the moment a regularization of the arising divergences (cf. below) by a high-momentum cut-off. In this way we also avoid a potential triviality problem (in four dimensions) of the quartic interactions.

To proceed in analogy with Sec. 3, we consider the *time-evolution* problem for the quantum field theory defined by eqs. (86,87) starting with given initial field configurations. In general, this can only be done in some approximation, e.g. ordinary perturbation theory for sufficiently small couplings in connection with any formalism able to handle time-dependent (non-equilibrium) systems such as (semi-classical) transport theory [31] or the Schwinger-Keldysh formalism (see e.g. Ref. [32] and references therein). Since the interesting results of Sec. 3 depend crucially on the short-time *strong-coupling* limit, cf. Sec. 3.2 and (63), in particular, and we anticipate the strong coupling at small four-momentum transfer in QCD to be most important for the environment-induced decoherence effects, we need a calculational scheme which allows us to consider the corresponding limit here.

## 4.2 Variational Approach to Time-evolution in Quantum Field Theory

It seems most efficient to employ the time-dependent variational principle of Dirac [15, 16] which is equivalent to the *functional Schrödinger equation* describing the full

dynamics of a generic field  $\varphi$  in the Schrödinger picture,

$$i\partial_t \Psi[\varphi; t] = H[\hat{\pi}, \varphi] \Psi[\varphi; t] \equiv \int d^d x \left\{ -\frac{1}{2} \frac{\delta^2}{\delta \varphi^2} + \frac{1}{2} (\nabla \varphi)^2 + \mathcal{V}(\varphi) \right\} \Psi[\varphi; t] , \quad (89)$$

where  $\Psi[\varphi; t] \equiv \langle \varphi | \Psi(t) \rangle$  denotes the wave functional in the  $\varphi$ -representation, which corresponds to a wave function  $\psi(x, t) \equiv \langle x | \psi(t) \rangle$  for a one-dimensional quantum-mechanical system, and  $\hat{\pi} = -i\delta/\delta\varphi$  represents the canonical momentum operator conjugate to the field (“coordinate”)  $\varphi$ . The dynamics is determined by the hamiltonian  $H$ . In this context the completeness and inner product relation, respectively, involve functional integrals instead of ordinary ones (orthogonality needs a  $\delta$ -functional),

$$\langle \Psi_1(t) | \Psi_2(t) \rangle \equiv \int \mathcal{D}\varphi \langle \Psi_1(t) | \varphi \rangle \langle \varphi | \Psi_2(t) \rangle = \int \mathcal{D}\varphi \Psi_1^*[\varphi; t] \Psi_2[\varphi; t] , \quad (90)$$

which induces

$$\langle \Psi(t) | O(\hat{\pi}, \varphi) | \Psi(t) \rangle = \int \mathcal{D}\varphi \Psi^*[\varphi; t] O(-i\frac{\delta}{\delta\varphi}, \varphi) \Psi[\varphi; t] , \quad (91)$$

i.e. the evaluation of expectation values of functions of operators corresponding to the usual quantum-mechanical formula

$$\langle \psi(t) | O(\hat{\pi}, x) | \psi(t) \rangle = \int dx \psi^*(x, t) O(-i\frac{d}{dx}, x) \psi(x, t) .$$

Furthermore, note that eq. (91) can be rewritten in terms of a density functional operator  $\hat{\rho}$ ,

$$\langle \Psi(t) | O(\hat{\pi}, \hat{\varphi}) | \Psi(t) \rangle = \int \mathcal{D}\varphi \langle \varphi | \Psi(t) \rangle \langle \Psi(t) | O(\hat{\pi}, \varphi) | \varphi \rangle \equiv \text{Tr } \hat{\rho}(t) O(\hat{\pi}, \hat{\varphi}) , \quad (92)$$

which is again analogous to the usual result, cf. eqs. (1,16,17), for example. Finally, we may state the *variational principle* [30, 33],

$$\frac{\delta \Gamma[\Psi]}{\delta \Psi} = 0 , \quad \text{for all } \Psi \text{ with } \langle \Psi(t) | \Psi(t) \rangle = 1 , \quad (93)$$

$$\text{and } \Gamma[\Psi] \equiv \int dt \langle \Psi(t) | [i\partial_t - H] | \Psi(t) \rangle , \quad (94)$$

i.e. requiring the *effective action*  $\Gamma$  defined in eq. (94) to be stationary against arbitrary variations of the normalized wave functional  $\Psi$ , which vanish at  $t \rightarrow \pm\infty$ , is equivalent to the exact functional Schrödinger equation, eq. (89) above. With the variational principle in hand, eqs. (93,94), one can solve the time-evolution problem in quantum field theory approximately by restricting the variation of the wave functional  $\Psi$  to a subspace of the full Hilbert space, which means by choosing a suitably parametrized trial wave functional.

In the following we choose to work with most general *gaussian trial wave functionals*. For the generic field  $\varphi$  it is defined by

$$\Psi_G[\varphi; t] \equiv N(t) \exp \left\{ -[\varphi - \bar{\varphi}(t)] \left[ \frac{1}{4}G^{-1}(t) - i\Sigma(t) \right] [\varphi - \bar{\varphi}(t)] + i\bar{\pi}(t)[\varphi - \bar{\varphi}(t)] \right\} , \quad (95)$$

where here and henceforth we use a shorthand notation suppressing integrations over spatial variables, whenever they are obvious. For example,

$$\begin{aligned} \varphi \mathbf{G}^{-1}(\mathbf{t}) \bar{\varphi}(\mathbf{t}) &\equiv \int d^d x d^d y \varphi(x) G^{-1}(x, y, t) \bar{\varphi}(y, t) , \\ \bar{\pi}(t) \varphi &\equiv \int d^d x \bar{\pi}(x, t) \varphi(x) , \quad \text{and} \quad \text{tr} \Sigma(t) \equiv \int d^d x \Sigma(x, x, t) . \end{aligned} \quad (96)$$

The normalization factor  $N$  can be easily calculated (for symmetric and positive-definite  $G$ ) according to eq. (90),

$$1 = \int \mathcal{D}\varphi \Psi_G^*[\varphi; t] \Psi_G[\varphi; t] \longrightarrow N(t) = (\mathcal{N} \det G(t))^{-1/4} , \quad (97)$$

which is a useful result for further calculations employing  $\Psi_G$  ( $\mathcal{N}$  is an infinite constant which can be omitted in the following). The meaning of the variational parameter functions  $\bar{\varphi}$ ,  $\bar{\pi}$ ,  $G$ , and  $\Sigma$  follows from [30, 33]:

$$\langle \varphi(x) \rangle_G = \bar{\varphi}(x, t) , \quad \langle -i \frac{\delta}{\delta \varphi(x)} \rangle_G = \bar{\pi}(x, t) , \quad (98)$$

$$\langle \varphi(x) \varphi(y) \rangle_G = \bar{\varphi}(x, t) \bar{\varphi}(y, t) + G(x, y, t) , \quad (99)$$

$$\langle i\partial_t \rangle_G = \bar{\pi}(t) \dot{\bar{\varphi}}(t) - \text{tr} [\dot{\Sigma}(t) G(t)] , \quad (100)$$

where the operator expectation values  $\langle \dots \rangle_G$  are calculated according to eq. (91) with  $\Psi_G$ . (There is a trivial error common to Refs. [30, 33] in the sign of the term  $\propto \bar{\pi}$  in the definition of the trial wave functional.) Thus, the trial wave functional  $\Psi_G$ , eq. (95), is a gaussian centered at  $\bar{\varphi}$  with a width  $G$ ;  $\bar{\pi}$  and  $\Sigma$  are “conjugate momenta” for  $\bar{\varphi}$  and  $G$ <sup>8</sup>, respectively. We anticipate that the application of a gaussian trial wave functional amounts to a description of the field theory under consideration in terms of *coupled equations for one- and two-point Wightman functions* (“TDHF”, time-dependent Hartree-Fock approximation). The equivalence with the Cornwall-Jackiw-Tomboulis generating functional (effective action) for two-particle irreducible graphs was demonstrated by those authors in Ref. [34] for energy eigenstates of the field.

Next, we evaluate the effective action, eq. (94), with the trial wave functional, eq. (95), to obtain:

$$\Gamma[\Psi_G] = \int dt \left\{ [\bar{\pi} \dot{\bar{\varphi}} - \frac{1}{2} \bar{\pi}^2 - \frac{1}{2} (\nabla \bar{\varphi})^2 - \mathcal{V}(\bar{\varphi})] \right.$$

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<sup>8</sup>This is obvious after a partial integration of the last term on the r.h.s. of eq. (100), which can be performed in the evaluation of the effective action, see eq. (101), for example.

$$\begin{aligned}
& + \text{tr} \left[ \Sigma \dot{G} - 2\Sigma G \Sigma - \frac{1}{8} G^{-1} + \frac{1}{2} \nabla^2 G \right] - \frac{1}{2!} \int d^3x \mathcal{V}^{(2)}(\bar{\varphi}) G(x, x) \\
& - \frac{3}{4!} \mathcal{V}^{(4)}(\bar{\varphi}) \int d^3x G^2(x, x) \} \quad , \quad (101)
\end{aligned}$$

where we suppressed spatial integrations as in (96) where possible and the time-dependence everywhere,  $\nabla^2$  acts only on either one of the two arguments of  $G$ ,  $\mathcal{V}^{(n)}(\bar{\varphi}) \equiv d^n \mathcal{V}(\bar{\varphi}) / d\bar{\varphi}^n$ , and we assume the potential to be at most quartic. Note that the calculation of  $\langle \mathcal{V}(\varphi) \rangle_G$ , in particular, proceeds conveniently through a Taylor expansion at an intermediate step:

$$\langle \Psi_G | \mathcal{V}(\varphi) | \Psi_G \rangle = N^2 \int \mathcal{D}\varphi \left\{ \mathcal{V}(\bar{\varphi}) + \frac{1}{2!} \mathcal{V}^{(2)}(\bar{\varphi}) \varphi^2 + \frac{1}{4!} \mathcal{V}^{(4)}(\bar{\varphi}) \varphi^4 \right\} e^{-\frac{1}{2} \varphi G^{-1} \varphi} \quad , \quad (102)$$

where we used eqs. (91,95) and observe that only even powers in  $\varphi$  contribute. Employing eq. (97), one obtains the simple gaussian integrals needed here by a suitable functional differentiation w.r.t.  $G^{-1}$  (cf. also eqs. (120,121) below),

$$\left( \varphi^{2[4]}(x) \right)_G \equiv N^2 \int \mathcal{D}\varphi \varphi^{2[4]}(x) e^{-\frac{1}{2} \varphi G^{-1} \varphi} = 1[3] \cdot G^{1[2]}(x, x) \quad , \quad (103)$$

and, thus, produces the potential contributions to the above result, eq. (101). We remark that the terms in the first line on the r.h.s. of eq. (101) present the usual classical action, cf. the hamiltonian in eq. (89), whereas the terms in the second and last line are  $O(\hbar)$  and  $O(\hbar^2)$  quantum corrections, respectively. From the effective action the relevant coupled equations of motion are obtained by independent variations w.r.t. to the variational parameter functions  $\bar{\varphi}$ ,  $\bar{\pi}$ ,  $G$ , and  $\Sigma$ . This will be performed for our model in the following.

#### 4.3 Model-independent Entropy in TDHF Approximation

After the above short review and formal preparations we are finally ready to study the *scalar parton field theory* defined in eqs. (86,87) with the variational technique. We assume a most general gaussian trial wave functional in product form,

$$\begin{aligned}
\Psi_{12}[\phi_1, \phi_2; t] & \equiv N_{12}(t) \Psi_{G_1}[\phi_1; t] \Psi_{G_2}[\phi_2; t] \\
& \cdot \exp \left\{ -\frac{1}{2} [\phi_1 - \bar{\phi}_1(t)] [G_{12}(t) - i\Sigma_{12}(t)] [\phi_2 - \bar{\phi}_2(t)] \right\} \quad , \quad (104)
\end{aligned}$$

with the normalized gaussians on the r.h.s. as defined in eq. (95). Here, however, each of the variational parameter functions in the expression for  $\Psi_{G_j}[\phi_j; t]$  carries an index  $j$  and  $N_{12}$  denotes an additional normalization factor. The latter is necessary, since we included here an essential exponential describing possible *two-point correlations* between the parton field  $\phi_1$  and the gluonic environment field  $\phi_2$ . It is obtained from the normalization condition, cf. eq. (97),

$$1 = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \Psi_{12}^*[\phi_1, \phi_2; t] \Psi_{12}[\phi_1, \phi_2; t]$$

$$\begin{aligned}
&= (N_1 N_2 N_{12})^2 \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp \left\{ -\frac{1}{2} [\phi_1 G_1^{-1} \phi_1 + \phi_2 \{G_2^{-1} - G_{12} G_1 G_{12}\} \phi_2] \right\} \\
&= (N_1 N_2 N_{12})^2 \left( \det G_1^{-1} \right)^{-1/2} \left( \det \{G_2^{-1} - G_{12} G_1 G_{12}\} \right)^{-1/2} \\
&= N_{12}^2 (\det \{1 - G_{12} G_1 G_{12} G_2\})^{-1/2} \\
&\longrightarrow N_{12}(t) = (\det \{1 - G_1(t) G_{12}(t) G_2(t) G_{12}(t)\})^{1/4} , \tag{105}
\end{aligned}$$

where we suitably shifted the fields to reach the second equation, assume  $G_{1,2}$  to be *symmetric and positive definite* and similarly have to restrict the argument of the determinant in the final result to assure the existence of the functional integrals (dropping irrelevant constants as mentioned after eq. (97)). We remark that the “1” in eq. (105) represents a  $\delta$ -function (of the space coordinates), which will occur frequently in the following. Furthermore, note that  $N_{12}$  is symmetric under  $G_1 \leftrightarrow G_2$ , as it should be.

To appreciate the importance of the non-diagonal factor (in  $\phi_{1,2}$ ) in eq. (104), which involves parton-environment correlations, we calculate the *partonic density functional* according to the general definition discussed in Sec. 2, cf. eq. (6):

$$\hat{\rho}_{\mathcal{P}}(t) \equiv \text{Tr}_{\mathcal{G}} \hat{\rho}(t) = \text{Tr}_2 |\Psi_{12}(t)\rangle \langle \Psi_{12}(t)| , \tag{106}$$

which yields the “matrix elements”

$$\begin{aligned}
\langle \phi_1 | \hat{\rho}_{\mathcal{P}}(t) | \phi'_1 \rangle &\equiv \rho_{\mathcal{P}}[\phi_1, \phi'_1; t] = \int \mathcal{D}\phi_2 \Psi_{12}^*[\phi'_1, \phi_2; t] \Psi_{12}[\phi_1, \phi_2; t] \\
&= N_{12}^2(t) \Psi_{G_1}^*[\phi'_1; t] \Psi_{G_1}[\phi_1; t] \\
&\quad \cdot \int \mathcal{D}\phi_2 \Psi_{G_2}^*[\phi_2; t] \Psi_{G_2}[\phi_2; t] \exp \left\{ -X_1[\phi'_1, \phi_1; t] (\phi_2 - \bar{\phi}_2) \right\} \\
&= N_{12}^2(t) \Psi_{G_1}^*[\phi'_1; t] \Psi_{G_1}[\phi_1; t] \exp \left\{ \frac{1}{2} X_1[\phi'_1, \phi_1; t] G_2(t) X_1[\phi'_1, \phi_1; t] \right\} ,
\end{aligned} \tag{107}$$

with

$$X_1[\phi'_1, \phi_1; t] \equiv \frac{1}{2} [\phi'_1 - \bar{\phi}_1(t)] [G_{12}(t) + i\Sigma_{12}(t)] + \frac{1}{2} [\phi_1 - \bar{\phi}_1(t)] [G_{12}(t) - i\Sigma_{12}(t)] , \tag{108}$$

and where we used eq. (104), see also eqs. (92,95). In addition to  $G_{12}$ , cf. eq. (105), we now also assume  $\Sigma_{12}$  to be *symmetric*, such that  $X_1^t = X_1$  w.r.t. space coordinates. Obviously, *if the correlation functions  $G_{12}(t)$  and  $\Sigma_{12}(t)$  vanish*, then the partonic density functional  $\rho_{\mathcal{P}}$  is just the one of an independent scalar field in the *pure state*  $|\Psi_{G_1}(t)\rangle$ . According to our previous general considerations in Sec. 2, cf. also the application in Sec. 3.2, the partonic entropy has to vanish in this case.

We can verify this conclusion by a direct calculation (similar to the one above) in the case of the *linear entropy*, which was defined in eq. (19), using the parton density functional from eqs. (107,108):

$$S_{\mathcal{P}}^{lin} = 1 - \text{Tr}_{\mathcal{P}} \hat{\rho}_{\mathcal{P}}^2 = 1 - N_{12}^4 \det(A)^{-1/2} \det(A - B A^{-1} B)^{-1/2} , \tag{109}$$

where again, as in  $N_{12}$  from eq. (105), we have to assume that the arguments of the determinants are symmetric and *positive definite*. They are defined in terms of

$$\begin{aligned} A(t) &\equiv 1 - \frac{1}{2}G_1(t)G_{12}(t)G_2(t)G_{12}(t) + \frac{1}{2}G_1(t)\Sigma_{12}(t)G_2(t)\Sigma_{12}(t) , \\ B(t) &\equiv \frac{1}{2}G_1(t)G_{12}(t)G_2(t)G_{12}(t) + \frac{1}{2}G_1(t)\Sigma_{12}(t)G_2(t)\Sigma_{12}(t) . \end{aligned} \quad (110)$$

The above result can be further simplified, if we assume all two-point functions to be *translation invariant*. Then, after a Fourier transformation, the quantities appearing in the arguments of the determinants in eq. (109) can be handled like ordinary numbers, which yields:

$$\begin{aligned} S_{\mathcal{P}}^{lin}(t) &= 1 - \det \left( \frac{1 - G_1 G_{12} G_2 G_{12}}{1 + G_1 \Sigma_{12} G_2 \Sigma_{12}} \right)^{1/2} = 1 - \exp \left\{ \frac{1}{2} \text{Tr} \ln(\dots) \right\} \\ &= 1 - \exp \left\{ \frac{1}{2} V_d \int \frac{d^d k}{(2\pi)^d} \ln \left( \frac{1 - G_1(k, t) G_2(k, t) G_{12}^2(k, t)}{1 + G_1(k, t) G_2(k, t) \Sigma_{12}^2(k, t)} \right) \right\} , \end{aligned} \quad (111)$$

where  $V_d \equiv \int d^d x$  in  $d$  dimensions.

Note that for any translation invariant two-point function  $F$  here, with

$$F(x, y; t) = F(x - y; t) \equiv \int \frac{d^d k}{(2\pi)^d} e^{ik \cdot (x-y)} F(k; t) , \quad (112)$$

the following constraints,

$$F(k; t) = F(-k; t) , \quad \text{and} \quad F(k; t) > 0 , \quad (113)$$

are implied by the requirements of being symmetric and positive definite.

Before we continue, we want to draw attention to several remarkable features of the above result for the linear entropy, which seem to be of a rather general nature:

**I.** Neither the “mean fields”  $\bar{\phi}_{1,2}$ , nor their conjugate momenta  $\bar{\pi}_{1,2}$ , nor the imaginary parts  $\Sigma_{1,2}$  of the parton and gluonic environment two-point functions, respectively, contribute to the entropy, i.e. to  $\text{Tr} \hat{\rho}_{\mathcal{P}}^2$ . This result is formally true for *arbitrary powers*,  $\text{Tr} \hat{\rho}_{\mathcal{P}}^n$ , since the corresponding terms always cancel between successive factors of  $\hat{\rho}_{\mathcal{P}}$ , when evaluating the trace. Thus, we anticipate it to hold for the statistical entropy as well (cf. below).

**II.** As expected, the *entropy vanishes for vanishing correlations between partons and gluonic environment*, i.e.  $G_{12}(t) = \Sigma_{12}(t) = 0$  (independent subsystems).

**III.** If either one of the *widths* of the parton or environment wave functionals vanishes,  $G_1(t) \rightarrow 0$  or  $G_2(t) \rightarrow 0$  (cf. eqs. (95,104), then the entropy vanishes. Thus, if either subsystem is constrained to follow essentially the *classical equations of motion*, then there is *no entropy production* (cf. discussion at the end of Sec. 4.4).

**IV.** The above conclusions **I.** - **III.** and the result for the linear entropy as given in eq. (111) are *model-independent*. They hold for *any* scalar field theory of partons coupled

to gluonic environment modes independently of the specific form of the (renormalizable) interactions. They are, however, obtained here in the *TDHF approximation* as embodied in the wave functional of eq. (104).

If we want to learn in detail about the time-evolution of the system, i.e. of the variational parameter functions, then we have to specify the interactions. The dynamics of the system with the model interactions from Sec. 4.1 will be studied in Sec. 4.4.

It is worthwhile stressing the importance of parton-environment correlations one more time, since these are precisely the quantum correlations discussed after eq. (17) in Sec. 2. Thus, our considerations presently confirm the general idea that *quantum decoherence* in a subsystem, and therewith a measurable *entropy production* therein, is induced by interactions with a dynamically active environment.<sup>9</sup> The parton model serves as a study case par excellence, since *confinement* dynamically separates the total system into an almost directly observable hard parton subsystem plus an unobservable non-perturbative environment of soft gluon modes (cf. Sec. 4.1).

Finally, we want to present here also an approximate result for the *statistical entropy*, which will later be more closely related to observables, the particle multiplicity in high-energy reactions, in particular. With the general definition of the statistical entropy in eq. (18) we find,

$$\begin{aligned} S_{\mathcal{P}} &= - \sum_p c_p \ln c_p \equiv \langle - \ln c_p \rangle \geq - \ln \langle c_p \rangle \\ &= - \ln \sum_p c_p^2 = - \ln \text{Tr}_{\mathcal{P}} \hat{\rho}_{\mathcal{P}}^2 = - \ln (1 - S_{\mathcal{P}}^{lin}) , \end{aligned} \quad (114)$$

where we also used eq. (19) and the inequality results from Jensen's inequality for convex functions, which is the basis for many variational principles (see e.g. Ref. [23]). The equality sign holds, if the (sub-)system is in a pure state (vanishing entropy) or if all states are equally probable,  $c_p = \text{const}$ , which is the case for a thermalized system at asymptotically high temperature ( $T \rightarrow \infty$ ), cf. eq. (23) for example. Applying eq. (114) to eq. (111), we obtain:

$$S_{\mathcal{P}}(t) \geq -\frac{1}{2}V_d \int \frac{d^d k}{(2\pi)^d} \ln \left( \frac{1 - G_1(k, t)G_2(k, t)G_{12}^2(k, t)}{1 + G_1(k, t)G_2(k, t)\Sigma_{12}^2(k, t)} \right) . \quad (115)$$

The right hand side here should provide a useful approximation for the statistical entropy in our scalar model, if we are interested particularly in the initial phase of entropy production, i.e. the decoherence process in a strong interaction at high energy, starting from a pure quantum state.

#### 4.4 Effective Action and Equations of Motion

Our aim in this concluding section is to derive the effective action for the complex system of partons and gluonic environment from which we then obtain the relevant

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<sup>9</sup>Note that a *functional diagonalization* of the parton density functional, see eqs. (106) - (108), would amount to an explicit construction of the time-dependent *pointer states* (see Sec. 2) for the parton field.

TDHF equations of motion, which describe the time-evolution of the system, the partonic subsystem in particular.

First of all, it helps somewhat to simplify the following calculations, if we rewrite the partonic density matrix from eqs. (107,108). We find:

$$\begin{aligned}\langle \phi_1 | \hat{\rho}_{\mathcal{P}}(t) | \phi'_1 \rangle &\equiv \rho_{\mathcal{P}}[\phi_1, \phi'_1; t] \\ &= \tilde{\Psi}_{G_1}^*[\phi'_1; t] \tilde{\Psi}_{G_1}[\phi_1; t] \exp \{ Y_1^*[\phi'_1; t] G_2(t) Y_1[\phi_1; t] \} ,\end{aligned}\quad (116)$$

with

$$Y_1[\phi; t] \equiv \frac{1}{2}[\phi - \bar{\phi}_1] [G_{12}(t) - i\Sigma_{12}(t)] , \quad (117)$$

and where the *effective gaussian*  $\tilde{\Psi}_{G_1}$  is defined as before, however, with the following replacements:

$$\begin{aligned}N_1(t) &\longrightarrow \tilde{N}_1(t) \equiv N_1(t) N_{12}(t) , \\ G_1^{-1}(t) &\longrightarrow \tilde{G}_1^{-1}(t) \equiv G_1^{-1}(t) A(t) , \\ \Sigma_1(t) &\longrightarrow \tilde{\Sigma}_1(t) \equiv \Sigma_1(t) - \frac{1}{8}[\Sigma_{12}(t) G_2(t) G_{12}(t) + G_{12}(t) G_2(t) \Sigma_{12}(t)] ,\end{aligned}\quad (118)$$

see eq. (95) for the definition of the generic  $\Psi_G$  and (110) for  $A(t)$ . (Recall that products of two-point functions involve an integration over intermediate coordinates.) Not surprisingly, the changes presently induced in the parton density functional by the environment bear some distant similarity with what happened to the single-parton density matrix in Sec. 3.1, cf. eqs. (51) - (61). It is also worthwhile to appreciate the analogies and differences between the Feynman-Vernon approach for the non-relativistic toy model, eqs. (31) - (35), and the TDHF result for *any* two coupled scalar fields, eqs. (116) - (118) above.

Next, we want to calculate the analogues of eqs. (98) - (100) to see the changes caused by the environment interacting with the scalar parton field:

$$\langle \phi_1(x) \rangle \equiv \int \mathcal{D}\phi_1 \phi_1(x) \rho_{\mathcal{P}}[\phi_1, \phi_1; t] = \bar{\phi}_1(x, t) , \quad (119)$$

$$\langle -i \frac{\delta}{\delta \phi_1(x)} \rangle \equiv \int \mathcal{D}\phi'_1 \mathcal{D}\phi_1 \delta[\phi'_1 - \phi_1] (-i) \frac{\delta}{\delta \phi_1(x)} \rho_{\mathcal{P}}[\phi_1, \phi'_1; t] = \bar{\pi}_1(x, t) .$$

Thus, there is no change in the mean fields here, as expected. However,

$$\begin{aligned}\langle \phi_1(x) \phi_1(y) \rangle &\equiv \int \mathcal{D}\phi_1 \phi_1(x) \phi_1(y) \rho_{\mathcal{P}}[\phi_1, \phi_1; t] \\ &= (N_1 N_{12})^2 \int \mathcal{D}\phi_1 [\phi_1(x) + \bar{\phi}_1(x, t)] [\phi_1(y) + \bar{\phi}_1(y, t)] \\ &\quad \cdot \exp \left\{ -\frac{1}{2} \phi_1 G_1^{-1} [A - B] \phi_1 \right\} \\ &= [\bar{\phi}_1(x, t) \bar{\phi}_1(y, t)] - 2(N_1 N_{12})^2 \frac{\delta}{\delta G_1^{-1} [A - B]|_{(x, y)}} (N_1 N_{12})^{-2} \\ &= [\bar{\phi}_1(x, t) \bar{\phi}_1(y, t)] + [A(t) - B(t)]^{-1} G_1(t)|_{(x, y)} ,\end{aligned}\quad (120)$$



where we used eqs. (97,105), and eqs. (110); we suitably shifted the field to obtain the second equation and employed (for  $M$  symmetric)

$$\frac{\delta}{\delta M(x, y)} \det M = \frac{\delta}{\delta M(x, y)} \exp\{\text{tr} \ln M\} = M^{-1}(x, y) \det M, \quad (121)$$

to obtain the final result. Obviously, the correlations with the environment modify the parton two-point function in a non-trivial way. Note that the relevant factor  $[A - B]^{-1}$  could be expanded in a *geometric series*. We remark that equations which are formally identical to (119) - (120) hold also for the field  $\phi_2$ . Due to the formal symmetry of the wave functional, eq. (104), they are obtained by simply exchanging the indices everywhere (" $1 \leftrightarrow 2$ "), which denote the parton and gluonic background fields, respectively. Furthermore, we obtain:

$$\begin{aligned} \langle i\partial_t \rangle &\equiv \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \Psi_{12}^*[\phi_1, \phi_2; t] (i\partial_t) \Psi_{12}[\phi_1, \phi_2; t] \\ &= \left\{ \bar{\pi}_1(t) \dot{\phi}_1(t) - \text{tr} \left[ \dot{\Sigma}_1(t) [A(t) - B(t)]^{-1} G_1(t) \right] \right\} + \{ "1 \leftrightarrow 2" \} \\ &\quad - \frac{1}{2} \text{tr} \left[ \dot{\Sigma}_{12}(t) \langle [\phi_1 - \bar{\phi}_1(t)] [\phi_2 - \bar{\phi}_2(t)] \rangle \right], \end{aligned} \quad (122)$$

where we employed eq. (120). Equation (122) generalizes eq. (100) by adding up separate contributions from both fields and a new term entirely due to the interaction between them. Consequently, a similar structure will arise in the evaluation of the total effective action below. We also find:

$$\begin{aligned} \langle [\phi_1(x) - \bar{\phi}_1(x, t)] [\phi_2(x) - \bar{\phi}_2(x, t)] \rangle &= 2N_{12}^{-1} \frac{\delta}{\delta G_{12}(x, y)} N_{12} \\ &= -\frac{1}{2} \{ G_2(t) G_{12}(t) [A(t) - B(t)]^{-1} G_1(t) + [A(t) - B(t)]^{-1} G_1(t) G_{12}(t) G_2(t) \}_{(x, y)}^S \\ &= -[A(t) - B(t)]^{-1} G_1(t) G_2(t) G_{12}(t)|_{(x, y)}, \end{aligned} \quad (123)$$

where the index  $S$  indicates full symmetrization w.r.t.  $G_1 \leftrightarrow G_2$  and separately  $x \leftrightarrow y$ . This symmetry stems from the symmetry of  $G_{12}$  and eq. (105), which is frequently used in the course of the calculations here with  $N_{12}^4 = \det(A - B)$ . For *translation invariant two-point functions* the last equality in (123) follows. This simplifying assumption will *always* be made henceforth.

As a final ingredient for the evaluation of the effective action we need

$$\begin{aligned} &\langle [\phi_1(x) - \bar{\phi}_1(x, t)]^2 [\phi_2(x) - \bar{\phi}_2(x, t)]^2 \rangle \\ &= (N_1 N_2 N_{12})^2 \left\{ 2 \cdot \frac{\delta^2}{\delta G_{12}(x, x)^2} + 4 \frac{\delta^2}{\delta G_1^{-1}(x, x) \delta G_2^{-1}(x, x)} \right\} (N_1 N_2 N_{12})^{-2} \\ &= 2 \langle [\phi_1 - \bar{\phi}_1] [\phi_2 - \bar{\phi}_2]_{(x)} \rangle^2 + \langle [\phi_1 - \bar{\phi}_1]_{(x)}^2 \rangle \langle [\phi_2 - \bar{\phi}_2]_{(x)}^2 \rangle \\ &\quad + 2G_{12}(x, x) \cdot [A - B]^{-2} G_1^2 G_2^2 G_{12}|_{(x, x)} + 2G_1 G_2 G_{12}|_{(x, x)} \cdot [A - B]^{-2} G_1 G_2 G_{12}|_{(x, x)} \end{aligned}$$

$$\begin{aligned}
& + \left\{ G_1(x, x) \cdot [A - B]^{-1} G_2|_{(x,x)} + \frac{1}{2} G_1(x, x) \cdot [A - B]^{-2} G_1 G_2^2 G_{12}^2|_{(x,x)} \right. \\
& \left. + \frac{1}{2} G_1 G_2^2 G_{12}^2|_{(x,x)} \cdot [A - B]^{-2} G_1|_{(x,x)} \right\} + \{ "1 \leftrightarrow 2" \} , \quad (124)
\end{aligned}$$

i.e. an expectation value with all four fields at the same point (the factor 2 in the first equation is a symmetry factor like the factor 3 in eq. (103)); we made use of eqs. (120,123) to rewrite terms as “disconnected parts” in the first line in the second equation and remark that the above result would look even worse without a final simplification due to translation invariance (intermediate spatial integrations are suppressed as before and the obvious time dependence is omitted).

To calculate the *effective action*, eq. (94), for the scalar field theory defined by eqs. (86,87) in TDHF approximation we proceed in analogy with Sec. 4.2 and evaluate various contributions separately:

$$\Gamma[\Psi_{12}] \equiv \int dt \langle \Psi_{12}(t) | [i\partial_t - H] | \Psi_{12}(t) \rangle = \int dt \{ \langle i\partial_t \rangle - \langle H_1 \rangle - \langle H_2 \rangle - \langle V_{12} \rangle \} . \quad (125)$$

First of all,  $\langle i\partial_t \rangle$  was evaluated already in eq. (122) together with eq. (123). Secondly, similarly as the corresponding terms (without time-derivatives) in eq. (101) before, we obtain after a lengthy calculation ( $j = 1, 2$ ):

$$\begin{aligned}
\langle H_j \rangle & \equiv \int d^d x \langle \Psi_{12}(t) | \left[ -\frac{1}{2} \frac{\delta^2}{\delta \phi_j^2} + \frac{1}{2} (\nabla \phi_j)^2 + v_j(\phi_j) \right] | \Psi_{12}(t) \rangle \\
& = \frac{1}{2} \bar{\pi}_j^2 + \frac{1}{2} (\nabla \bar{\phi}_j)^2 + v_j(\bar{\phi}_j) \\
& \quad - \text{tr} \left[ -2 \tilde{\Sigma}_j [A - B]^{-1} G_j \tilde{\Sigma}_j - \frac{1}{8} G_j^{-1} [A + B] + \frac{1}{2} [A - B]^{-1} \nabla^2 G_j \right] \\
& \quad + \int d^3 x \left[ \frac{1}{2!} v_j^{(2)}(\bar{\phi}_j) \cdot [A - B]^{-1} G_j|_{(x,x)} + \frac{3}{4!} v_j^{(4)}(\bar{\phi}_j) \cdot ([A - B]^{-1} G_j|_{(x,x)})^2 \right] , \quad (126)
\end{aligned}$$

where the same comments as after eq. (101) apply and we made use of equations analogous to (102,103) which hold for the present case. Furthermore, we employed eq. (120) and recalled the definition of  $\tilde{\Sigma}_1$  in eq. (118), while  $\tilde{\Sigma}_2$  follows with the familiar exchange “1  $\leftrightarrow$  2”. We remark that eq. (126) correctly yields the corresponding terms in eq. (101) for *vanishing correlations* ( $A = 1$ ,  $B = 0$ ) between the fields  $\phi_1$  and  $\phi_2$ , i.e. for independent subsystems.

Next, we have to calculate the contribution  $\langle V_{12} \rangle$  from the mutual interaction of the fields. Employing the appropriate Taylor expansion for a general potential, which is at most quartic in the fields (cf. also eqs. (102,103)), and keeping only non-vanishing terms yields:

$$\begin{aligned}
\langle V_{12} \rangle & \equiv \int d^d x \langle \Psi_{12}(t) | V(\phi_1, \phi_2) | \Psi_{12} \rangle \\
& = V(\bar{\phi}_1, \bar{\phi}_2) + \frac{2}{2!} V^{(1,1)} \langle (\phi_1 - \bar{\phi}_1)(\phi_2 - \bar{\phi}_2) \rangle + \frac{6}{4!} V^{(2,2)} \langle (\phi_1 - \bar{\phi}_1)^2 (\phi_2 - \bar{\phi}_2)^2 \rangle \\
& \quad + \left\{ \frac{1}{2!} V^{(2,0)} \langle (\phi_1 - \bar{\phi}_1)^2 \rangle + \frac{1}{4!} V^{(4,0)} \langle (\phi_1 - \bar{\phi}_1)^4 \rangle \right\} + \{ "1 \leftrightarrow 2" \} , \quad (127)
\end{aligned}$$

where  $V^{(m,n)} \equiv \frac{d^m}{d\phi_1^m} \frac{d^n}{d\phi_2^n} V(\bar{\phi}_1, \bar{\phi}_2)$  and we omitted the overall  $\int d^d x$  in the last equation. The expectation values arising here have all been calculated before. In particular, we recall eq. (103), which applies for the terms in the last line with  $\varphi \rightarrow \phi_{1,2}$  and  $G^{-1} \rightarrow G_{1,2}^{-1} \cdot [A - B]$  after suitably shifting the fields; these terms obviously generate additional contributions of a similar (mean field dependent) structure as the potential terms  $\propto v_j^{(2),(4)}$  in eq. (126). The most interesting terms besides the classical contribution in the second to last line in eq. (127) are obtained explicitly by inserting eqs. (123,124).

This completes our derivation of the Cornwall-Jackiw-Tomboulis type *effective action* [34] for any system of *two coupled scalars fields* with at most quartic interactions. We implemented Dirac's time-dependent variational principle, eqs. (93,94), with the help of a most general gaussian trial wave functional, see eq. (104). Now, combining eqs. (122) - (127), we obtain the final result (with  $\hbar$ 's inserted):

$$\begin{aligned}
\Gamma[\Psi_{12}] = & \int dt \left\{ \sum_{j=1,2} \left\{ \bar{\pi}_j \dot{\bar{\phi}}_j - \frac{1}{2} \bar{\pi}_j^2 - \frac{1}{2} (\nabla \bar{\phi}_j)^2 - v_j(\bar{\phi}_j) \right. \right. \\
& + \hbar \operatorname{tr} [ \Sigma_j \dot{\bar{G}}_j - 2 \tilde{\Sigma}_j \bar{G}_j \tilde{\Sigma}_j - \frac{1}{8} G_j^{-1} [A + B] + \frac{1}{2} \nabla^2 \bar{G}_j ] \\
& - \frac{\hbar}{2!} \langle \{ v_j^{(2)} + V_j^{(2)} \} \rangle \operatorname{tr} \bar{G}_j - \frac{\hbar^2}{V_d} \frac{3}{4!} \{ v_j^{(4)} + V_j^{(4)} \} (\operatorname{tr} \bar{G}_j)^2 \} \\
& + \frac{\hbar}{2} \operatorname{tr} [ \dot{\Sigma}_{12} \bar{G}_1 \bar{G}_2 \bar{G}_{12} ] - V(\bar{\phi}_1, \bar{\phi}_2) + \hbar \frac{2}{2!} \langle V^{(1,1)} \rangle \operatorname{tr} [\bar{G}_1 \bar{G}_2 \bar{G}_{12}] \\
& - \frac{\hbar^2}{V_d} \frac{6}{4!} V^{(2,2)} \left[ 2 (\operatorname{tr} \bar{G}_1 \bar{G}_2 \bar{G}_{12})^2 + \operatorname{tr} \bar{G}_1 \operatorname{tr} \bar{G}_2 \right. \\
& + 2 \operatorname{tr} G_{12} \operatorname{tr} [G_1^2 G_2^2 G_{12} [A - B]^{-2}] + 2 \operatorname{tr} [G_1 G_2 G_{12}] \operatorname{tr} [G_1 G_2 G_{12} [A - B]^{-2}] \\
& + \{ \operatorname{tr} G_1 \operatorname{tr} \bar{G}_2 + \frac{1}{2} \operatorname{tr} G_1 \operatorname{tr} [G_1 G_2^2 G_{12}^2 [A - B]^{-2}] \\
& \left. \left. + \frac{1}{2} \operatorname{tr} [G_1 G_2^2 G_{12}^2] \operatorname{tr} [G_1 [A - B]^{-2}] \right\} + \{ "1 \leftrightarrow 2" \} \right] \Big\} , \tag{128}
\end{aligned}$$

with  $v_j^{(n)} \equiv d^n v_j(\bar{\phi}_j) / d\bar{\phi}_j^n$ ,  $V_j^{(n)} \equiv d^n V(\bar{\phi}_1, \bar{\phi}_2) / d\bar{\phi}_j^n$ , and  $V^{(m,n)}$  as defined after eq. (127). We also use the abbreviations  $\bar{G}_{12} \equiv G_{12}[A - B]$ ,  $\bar{G}_j \equiv [A - B]^{-1} G_j$ , and  $\nabla^2$  acts on either one of the two formal arguments of  $\bar{G}_j$ ;  $\tilde{\Sigma}_1$  was defined in (118),  $\tilde{\Sigma}_2$  follows by “1  $\leftrightarrow$  2”, and  $A, B$  are given in eqs. (110). We remark that due to the assumption of *translation invariance* of the two-point functions their position in a product is irrelevant, if they are connected by integrations over intermediate coordinates.<sup>10</sup> Furthermore, we employed eqs. (136) from the Appendix and introduced

<sup>10</sup>Implementing translation invariance into the effective action does not interfere with taking one further functional derivative as in deriving equations of motion, if proper care is taken of the “zero mode”, see the Appendix.

the *spatial average* of a function  $f$ ,  $\langle f \rangle \equiv \int d^d x f(x)/V_d$ , with  $V_d \equiv \int d^d x$  denoting the volume of the region of integration. Of course, all one- and two-point functions in eq. (128) are time-dependent, which we suppressed.

We conclude this section by deriving the corresponding TDHF equations of motion for the *ten variational parameter functions* entering the effective action in eq. (128). In particular, these equations describe the time-evolution of our *parton model field theory* defined by eqs. (86,87) together with eqs. (88):

$$\frac{\delta \Gamma}{\delta \bar{\pi}_j(x, t)} = 0 \implies \dot{\bar{\phi}}_j(x, t) = \bar{\pi}_j(x, t) . \quad (129)$$

$$\frac{\delta \Gamma}{\delta \Sigma_j(x, y, t)} = 0 \implies \dot{\bar{G}}_j(x, y, t) = 4\tilde{\Sigma}_j \bar{G}_j|_{(x, y; t)} . \quad (130)$$

$$\begin{aligned} \frac{\delta \Gamma}{\delta \Sigma_{12}(x, y, t)} = 0 \implies \partial_t \left( \bar{G}_1 \bar{G}_2 \bar{G}_{12} \right)_{(x, y; t)} &= 2(\tilde{\Sigma}_1 + \tilde{\Sigma}_2) \bar{G}_1 \bar{G}_2 \bar{G}_{12}|_{(x, y; t)} \\ &\quad - \frac{1}{2}(G_1 + G_2) \Sigma_{12}|_{(x, y; t)} . \end{aligned} \quad (131)$$

$$\begin{aligned} \frac{\delta \Gamma}{\delta \bar{\phi}_j(x, t)} = 0 \implies \dot{\bar{\pi}}_j(x, t) &= \ddot{\bar{\phi}}_j(x, t) = \nabla^2 \bar{\phi}_j(x, t) - v_j^{(1)} - V_j^{(1)} \\ &\quad - \frac{\hbar}{2} \{v_j^{(3)} + V_j^{(3)}\} \bar{G}_j(x, x, t) + \hbar \frac{d}{d\phi_j} V^{(1,1)} \bar{G}_1 \bar{G}_2 \bar{G}_{12}|_{(x, x; t)} , \end{aligned} \quad (132)$$

where we used eq. (129) to indicate that  $\bar{\pi}_j$  can be easily eliminated from the set of equations. Apart from eq. (131) and the last term in eq. (132), the above equations still have a structure similar to the ones which can be obtained from eq. (101) in the case of a single scalar field [30, 33]. Note that eq. (131) can be explicitly written as an equation for  $\dot{\bar{G}}_{12}$ , eliminating  $\dot{\bar{G}}_j$  by eq. (130).

We remark that the equations following by variation w.r.t. the two-point functions  $\Sigma_j$  and  $\Sigma_{12}$ , in particular, could be derived by directly applying the respective functional derivatives on the effective action, since *all* contributions arise in this case from trace terms in eq. (128). For the other two-point functions,  $G_j$  and  $G_{12}$ , there are terms with potential insertions. Here one has to be more careful, taking into account that the two-point functions actually depend only on *one* variable due to translation invariance. The relevant formulae are given in the Appendix.

Then, we obtain another pair of equations which generalize the corresponding one for a single scalar field ( $j' \neq j$ ,  $j = 1, 2$ ):

$$\begin{aligned} \frac{\delta \Gamma}{\delta G_j(x, y, t)} = 0 \implies \\ \dot{\Sigma}_j(x, y, t) &= [-2\tilde{\Sigma}_j \tilde{\Sigma}_j + \frac{1}{8} \bar{G}_j^{-2} + \frac{1}{2} G_{j'} G_{12} \dot{\Sigma}_{12} + G_{j'} \tilde{\Sigma}_{j'} \bar{G}_{12} \Sigma_{12}]_{(x, y; t)} \\ &\quad + \left[ \frac{1}{2} \nabla^2 - \frac{1}{2} \langle \{v_j^{(2)} + V_j^{(2)}\} \rangle - \frac{\hbar}{V_d} \frac{1}{4} \{v_j^{(4)} + V_j^{(4)}\} \text{tr } \bar{G}_j \right]_{(t)} \cdot \delta(x - y) \end{aligned}$$

$$\begin{aligned}
& + \langle V^{(1,1)} \rangle G_{j'} G_{12} |_{(x,y;t)} - \frac{\hbar}{V_d} \frac{1}{4} V^{(2,2)} \left[ 4 \operatorname{tr} [\bar{G}_1 \bar{G}_2 \bar{G}_{12}] \cdot G_{j'} G_{12} \right. \\
& + \operatorname{tr} G_{j'} \cdot [A - B]^{-1} + \frac{1}{2} \operatorname{tr} G_j \cdot G_j^2 G_{12}^2 F + \operatorname{tr} [\bar{G}_{j'} [A - B]^{-1}] \cdot G_1 G_2 \bar{G}_{12}^2 \\
& + \operatorname{tr} [G_j^2 G_{j'} G_{12}^2] \cdot G_j^2 G_{12}^2 [A - B]^{-1} + \frac{1}{2} \operatorname{tr} [\bar{G}_j [A - B]^{-1}] \cdot G_{j'}^2 \bar{G}_{12}^2 \\
& + 4 \operatorname{tr} G_{12} \cdot \bar{G}_j G_{j'}^2 G_{12} + 2 \operatorname{tr} [\bar{G}_1 \bar{G}_2 G_{12}] \cdot G_{j'} G_{12} [A - B]^2 \\
& + 2 \operatorname{tr} [G_1 G_2 G_{12}] \cdot G_{j'} G_{12} F + \frac{1}{2} \operatorname{tr} [G_j \bar{G}_{j'}^2 G_{12}^2] \cdot [A - B]^2 \\
& \left. + \frac{1}{2} \operatorname{tr} [G_j G_{j'}^2 G_{12}^2] \cdot F + \operatorname{tr} \bar{G}_{j'} \cdot (\hat{1} + [A - B]^2) \right]_{(x,y;t)}, \quad (133)
\end{aligned}$$

where we used formulae from the Appendix and inserted the abbreviations introduced after eq. (128) and  $F \equiv [1 + G_1 G_2 G_{12}^2] / [1 - G_1 G_2 G_{12}^2]$  to write the result as compact as possible. For the same reason we collected the arguments from individual terms and wrote  $\hat{1}$  for a  $\delta$ -function in the last line. At last, we find:

$$\begin{aligned}
\frac{\delta \Gamma}{\delta G_{12}(x, y, t)} = 0 \implies \\
\dot{\Sigma}_{12}[\hat{1} + G_1 G_2 G_{12}^2] |_{(x,y;t)} = 4 \sum_{j=1,2} \left[ -\frac{1}{2} \tilde{\Sigma}_j \Sigma_{12} [A - B] + (\dot{\Sigma}_j + 2 \tilde{\Sigma}_j^2) G_j G_{12} \right. \\
- \left[ \frac{1}{2} \nabla^2 - \frac{1}{2} \langle \{v_j^{(2)} + V_j^{(2)}\} \rangle - \frac{\hbar}{V_d} \frac{1}{4} \{v_j^{(4)} + V_j^{(4)}\} \operatorname{tr} \bar{G}_j \right]_{(t)} \cdot G_j G_{12} \Big|_{(x,y;t)} \\
- 2 \langle V^{(1,1)} \rangle [\hat{1} + G_1 G_2 G_{12}^2] |_{(x,y;t)} + \frac{\hbar}{V_d} V^{(2,2)} \left[ 2 \operatorname{tr} [\bar{G}_1 \bar{G}_2 \bar{G}_{12}] \cdot [\hat{1} + G_1 G_2 G_{12}^2] \right. \\
+ \operatorname{tr} [\bar{G}_1^2 \bar{G}_2^2 \bar{G}_{12} [A - B]] \cdot \bar{G}_1^{-1} \bar{G}_2^{-1} + \operatorname{tr} [\bar{G}_1 \bar{G}_2 G_{12}] \cdot [A - B]^2 \\
+ \operatorname{tr} G_{12} \cdot G_1 G_2 (4[A - B]^{-1} - 3) + \operatorname{tr} [G_1 G_2 G_{12}] \cdot (4[A - B]^{-1} - 3) \\
+ \{ \operatorname{tr} \bar{G}_1 \cdot G_2 G_{12} + \operatorname{tr} G_1 \cdot G_2 G_{12} (\hat{1} + \frac{1}{2} F) + \operatorname{tr} [G_1 G_2^2 G_{12}^2] \cdot \bar{G}_1 G_{12} \\
\left. + \frac{1}{2} \operatorname{tr} [\bar{G}_1 [A - B]^{-1}] \cdot G_2 \bar{G}_{12} [A - B] \} + \{ \text{“}1 \leftrightarrow 2\text{”} \} \Big|_{(x,y;t)}, \quad (134)
\end{aligned}$$

where the same comments as after eq. (133) apply; in particular, note that  $\nabla^2$  acts on either one argument of the following term, as before.

Thus, with eqs. (129) - (134) we obtained a closed set of *ten coupled non-linear equations* which describe the time-evolution of any theory of *two coupled scalar fields* in terms of one- and two-point Wightman functions (TDHF approximation). They are local in time and of first order in time derivatives (see, however, eq. (132) for the elimination of  $\bar{\pi}_j$  and consequently  $\partial_t \bar{\phi}_j \rightarrow \partial_t^2 \bar{\phi}_j$ ).

Generally, one has to expect the usual ultraviolet divergences in terms like  $\operatorname{tr} G(t) = \int d^d x G(x, x)$  corresponding to expectation values of two fields at coinciding points. We assume them to be regulated by a high-momentum cut-off at present. Note that by their definition via the Momentum Space Mode Separation in Sec. 4.1 the gluonic environment modes are automatically regulated in this respect. Therefore, one could

essentially take over the separate and only necessary *renormalization of the scalar parton* sector from Refs. [30, 33]. We plan to come back to this feature in the sequel to the present work. There we also have to specify physical *initial conditions* for the set of parton-environment TDHF equations and study the restrictions imposed by renormalization.

In passing we remark that due to translation invariance eqs. (129) - (134) simplify considerably after a *Fourier transformation*. From multiple convolution type spatial integrals involving two-point functions there is always only one final momentum integration left over, cf. e.g. eqs. (111,115) where this was employed before. Incidentally, for any translation invariant two-point  $F$  function here, with

$$F(p; t) \equiv \int d^d x e^{-ip \cdot y} F(y; t) = \int d^d x e^{-ip \cdot y} F(x + \tfrac{1}{2}y, x - \tfrac{1}{2}y; t) , \quad (135)$$

the Fourier transform coincides with the non-covariant *Wigner function*. Therefore, after a straightforward Fourier transformation, our set of equations at the same time presents the *transport theory* [31] for any two coupled scalar fields in TDHF approximation. It is obtained here in the non-covariant form suitable for initial value problems and should allow interesting comparisons with other transport equations derived in weak-coupling perturbation theory.

Finally, we point out two special cases of the parton-environment TDHF equations, which are of interest in their own right:

(a) Setting  $\bar{\phi}_j \equiv 0 \equiv \bar{\pi}_j$ , i.e. *no classical mean fields*, the set of equations reduces to *six* coupled equations. This may be relevant for a short-lived system with no such fields in the initial state which is in some sense dominated by incoherent radiation, i.e., if there is no time to develop classical expectation values of the fields. It might also apply to a densely (in phase space) populated parton system, if (and only *IF*) the effect of such a “quasi high-temperature” state is to “melt” the classical fields.

(b) Conversely, if we constrain one part of the complex system to follow the classical equations of motion, i.e. to be completely *mean field dominated*, by setting e.g.  $G_2 \equiv 0 \equiv \Sigma_2$ , then the set of equations again reduces to only *six* coupled equations, since the dependence on the correlation functions  $G_{12}$  and  $\Sigma_{12}$  is automatically eliminated from the effective action, eq. (128). This is in agreement with our conclusions concerning *entropy production*, cf. eqs. (111,115) and point **III.** in Sec. 4.3, which is identically *zero* here. Obviously, this special case (i.e. the QCD analogue to be worked out) cannot be relevant for high-multiplicity events in strong interactions at high energy and the parton model as applied to such events, in particular.

We further discuss these issues in the following section.

## 5. Conclusions

At the outset we apologize to all practitioners and followers of the QCD parton model. We did not consider QCD partons and their gluonic environment yet. However, our considerations and the basic idea towards a solution of the about 40 years

old “*entropy puzzle*” in strong interactions at high energy are motivated by what we believe to be important features of QCD. This explains our abuse of the words “parton” and “gluonic” environment modes.

To begin with, throughout this work and in Sec. 2, in particular, we consider the von Neumann or *statistical entropy* defined in terms of the relevant density matrix, see e.g. eq. (18), the part of Sec. 3 following point (iii), and especially Sec. 4.3. For technical reasons we often employ the *linear entropy*, eq. (19), which we show to provide a lower bound on the statistical entropy, see eq. (114). Concerning *quantum* properties of the system to be characterized by the entropy, both definitions are equally valid for our purposes. In particular, both measure the *impurity* of the parton density matrix.

The core of the entropy problem is to try and understand how (an idealized example) two hadronic scattering in-states undergoing a hard interaction, i.e. a quantum mechanically *pure initial state*, can result in a high-multiplicity event corresponding to a highly impure (more or less thermal) density matrix on the parton level before hadronization.

Thus, attempts to associate the apparent entropy production with a “coarse graining” either in the phase space of the observed system or due to the finite resolution in any experimental measurement, which both may be useful to characterize derived “macroscopic” aspects of such reactions, seem to miss the point. There one gives up from the beginning the possibility to understand on a fundamental dynamical level how a *complex pure-state quantum system* can produce classical behaviour, i.e. an impure density (sub-)matrix with *decoherence* of associated (parton) wave functions and *entropy* production.

In Sec. 2 we introduce a convenient general framework in terms of *Schmidt* and *pointer states* to analyze the consequences of a dynamical separation of a complex (possibly strongly interacting) system into an observable (“partonic”) subsystem plus unobservable (“gluonic”) environment modes [7, 8, 9, 21]. Entropy production due to *environment-induced quantum decoherence* in the observable subsystem arises naturally, *IF* such a separation of the total “closed” system into an “open” subsystem and its environment is *dynamically* realized. There is no guarantee for this to happen and the existence of the associated (almost) classically behaving (partonic) pointer states, i.e. states the quantum superpositions of which are dynamically suppressed, has to be viewed as a particular feature of the system.

Such a major miracle seems to be installed in QCD [29]: for example, the fact that deep-inelastic scattering can be described accurately in terms of a hard scattering cross section and structure functions. The latter are *decoherent* one-particle *probability distributions*. Therefore, some “secret agent” has to effect this extremely efficient decoherence process, once a parton is knocked out of its coherent initial state wave function. Our point of view here does not conflict with the standard parton picture, in particular, the applicability of plane-wave states representing partons entering or leaving Feynman diagrams.<sup>11</sup> Any basis should do. However, the miracle consists in the fact that typically the initial state of a hadronic scattering reaction can be

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<sup>11</sup>I thank R. Baier for insisting with his questions about this.

described by structure functions which correspond to *diagonal* density matrices, i.e. without interference terms (which *always* can be achieved formally by the Schmidt decomposition procedure, however, only at *one instant of time*, cf. Sec. 2).

In terms of the split hamiltonian in eq. (21), the bound partonic initial state wave function diagonalizes  $\hat{H}_{\mathcal{P}} + \hat{H}_{\mathcal{PG}}$ . Once it is perturbed by an external (e.g. electromagnetic) interaction, the interaction with the gluonic environment starts to “rattle and shake” the perturbed wave function so strongly that it decays very fast into a decoherent superposition of states as reflected in a *structure function*.

We conjecture that gluonic modes (neglecting quarks for now) which are close to *constant field configurations* form an essentially unobservable sector of the total Hilbert space of a QCD system. The associated Momentum Space Mode Separation is discussed in more detail in Sec. 4.1. They act as an active environment on the observable<sup>12</sup> partonic sector, to which they are strongly coupled via low- $|Q|^2$  processes. Presently we explore this picture in two simple models.

In Sec. 3 we study a non-relativistic parton coupled to a gluonic environment in close analogy to an electron coupled to the quantized electromagnetic field [27, 28]. However, we deliberately change the spectral density of the environmental oscillators to be dominated in the infrared. We calculate the time-dependent parton density matrix by integrating out the environmental degrees of freedom exactly with the Feynman-Vernon influence functional technique [23, 26]. In a *short-time strong-coupling* limit, which has not been of interest in studies of quantum Brownian motion so far [26], we find the following results analytically (cf. fig. 1 and points (i) - (iii) in Sec. 3.2):

Gaussian parton wave packets experience *friction* and *localization* and their coherent *superpositions decohere*.

All effects are governed by a non-perturbative time-scale  $f_+^{-1} \ll 1\text{fm}/c$ , eq. (64). Thus, a parton following the classical trajectory with the center of its wave packet is slowly decelerated, which can be interpreted as being due to the emission of infrared gluons, i.e. the excitation of environment modes in this model. More surprising is the localization effect, i.e., the quantum mechanical spreading of the wave packet can be suppressed or even reversed depending on the choice of parameters. The initial wave function obtains an almost soliton like character through the strong interaction with the environment. Finally, considering the superposition of wave packets, we find that their decoherence actually happens in the short-time strong-coupling limit. Consequently, in accordance with the general considerations of Sec. 2, we “see” and can calculate how *entropy* is produced in this toy model on a short time scale.

All the above effects, if recovered in QCD eventually, seem to be very important e.g. for a justification of the classical cascade approach to multiple parton scattering [18]. Then, a parton propagating in space-time behaves essentially like a *classical particle* in between perturbative hard scatterings.

In Sec. 4 we set up a fairly general model of two coupled scalar fields representing partons and gluonic environment modes, respectively (see Sec. 4.1 for details). In this case, due to the non-linear interactions, the environment degrees of freedom

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<sup>12</sup>In the sense of parton-hadron duality or deep-inelastic scattering.



cannot be integrated out as before. Therefore, our main concern here is to formulate a tractable approach to the time-evolution problem in this quantum field theory, which is not restricted to perturbatively small couplings, in particular. We employ Dirac’s variational principle [15, 16], see Sec. 4.2, and describe the complex system in terms of its *ten one- and two-point Wightman functions* (TDHF approximation). This corresponds to the most general gaussian wave functional, eq. (104), in the field theory Schrödinger picture. In Sec. 4.4 we derive the related Cornwall-Jackiw-Tomboulis type *effective action* [34], eq. (128), governing the dynamics and obtain the equations of motion from it.

An attractive feature of having an explicit wave functional in hand is that it allows to calculate the *partonic density functional* here quite generally, eqs. (106) - (108) or eqs. (116) - (118), without having to solve the complicated dynamics first of all. Thus, we obtain in Sec. 4.3 a *model-independent* result for the *entropy* in terms of two-point functions, eqs. (111,115), which causes several interesting observations, see points **I.** - **IV.** there, which are presumably of a general nature:

1. If there are *no quantum correlations* between the partons and the gluonic environment, then there is *no parton decoherence* and *no entropy production*. Again, this is in accordance with the general considerations in Sec. 2.
2. If one assumes a strictly *classical gluonic environment*, i.e. following the classical equations of motion, then there is *no parton decoherence* and *no entropy production*.

This latter observation is in interesting contrast to recent attempts to explain entropy production and thermalization in QCD as being due to the non-perturbative chaotic dynamics of purely classical Yang-Mills fields, see Ref. [35] and references therein. There the above mentioned “coarse graining” of the classical phase space covered by an *ensemble* of identical systems<sup>13</sup> is necessary to deduce a *classical entropy*. The results are very suggestive as compared to high-temperature field theory. However, in both cases decoherence is put in by hand. Their relevance for the von Neumann entropy considered here and the most important *dynamical decoherence* phenomena in a *quantum* parton system, in particular, seems to be unclear at present.

Our results in Sec. 4 will be the starting point of the sequel to this work, where we plan to study physical initial conditions for the parton-environment system and their relation to model structure functions. Then, of course, the time-evolution of the system following the equations of motion, eqs. (129) - (134), will be most interesting to consider. Any solutions will provide the explicit time-dependence of entropy production according to eqs. (111,115), especially for early stages of a reaction.

Let us summarize our present point of view by saying that a parton appears like a parton only because it feels the gluonic environment which manifests its *strong* non-perturbative interactions caused by the running coupling on a short time scale ( $\ll 1\text{fm}/c$ ) through the induced decoherence properties of partons and the associated observable entropy production.

Finally, it seems worthwhile to mention a few other potential applications of our

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<sup>13</sup>Of course, this does *not* represent a *pure* quantum state and, thus, the resulting “thermalization” seems to be a very plausible consequence of the mixing property of a strongly chaotic system, i.e. the filamentation of the phase space occupied by the ensemble.

results from Sec. 4. They provide a first step to study the analogue of *quantum Brownian motion* [26] in the context of field theory. In particular, one may study the time-dependence of a Higgs model type phase transition under the influence of the interaction with a perturbing environment field. The possibility of such a *non-equilibrium phase transition* is contained already in our simplest specification of the interactions in eqs. (88). It has been another long-standing problem to study changes of signals, which are thought to characterize an equilibrium phase transition, when they come from an out-of-equilibrium system such as the early stage of a high-energy nuclear collision. Recently, following the suggestion of the possible formation of a disordered chiral condensate in relativistic heavy-ion collisions, it has been realized that the condensate field actually presents an *open system*, see Refs. [36] and numerous references therein, to which our methods can be applied.

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### Appendix

We list several formulae which are useful for the calculations in Sec. 4.4.<sup>14</sup> They are valid for any translation invariant symmetric two-point functions  $X, Y, Z$ , and any integrable function  $\mathcal{V}$  (in any dimension):

$$\begin{aligned}
I &\equiv \int dx \mathcal{V}(x) \cdot XYZ|_{(x,x)} \\
&= \int dx dx_1 dx_2 \mathcal{V}(x) X(x - x_1) Y(x_1 - x_2) Z(x_2 - x) \\
&= \int dx \mathcal{V}(x) \cdot XZY|_{(x,x)} \quad [\text{or any other permutation of } XYZ] \\
&= \int dx \mathcal{V}(x) \text{tr} [XYZ] / \text{vol} \quad ,
\end{aligned} \tag{136}$$

where  $\text{vol} \equiv \int dy$  denotes the volume of the region of integration. Then, functional derivatives w.r.t. one of the two-point functions can be calculated in two ways employing the *trace formula* in (136):

$$\frac{\delta}{\delta Z(z)} I = \int dx' \mathcal{V}(x') \frac{\delta}{\delta Z(z)} \text{tr} [XYZ] / \text{vol}$$

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<sup>14</sup>C. Wetterich helped with a remark here at the right moment.

$$\begin{aligned}
&= \int dx' \mathcal{V}(x') XY|_{(x,y)} \quad [\text{for } z \equiv x - y] \\
&= \int dx' \mathcal{V}(x') \frac{\delta}{\delta Z(x,y)} \text{tr} [XYZ] .
\end{aligned} \tag{137}$$

Dividing out the constant factor, one obtains:

$$\frac{\delta}{\delta Z(x-y)} \text{tr} [XYZ] = \text{vol} \cdot \frac{\delta}{\delta Z(x,y)} \text{tr} [XYZ] , \tag{138}$$

which clearly exhibits the “zero mode” factor connecting the functional derivative which takes translational invariance into account with the one which does not.

Finally, with the definitions of  $A, B$  and  $\bar{G}_j, \bar{G}_{12}$  given in eqs. (110) and after eq. (128), respectively, one finds the following simple results:

$$\text{tr} \left[ X \frac{\delta \bar{G}_j}{\delta G_j(x,y)} \right] = \frac{X}{[A-B]^2}|_{(x,y)} , \quad \text{tr} \left[ X \frac{\delta \bar{G}_{12}}{\delta G_{12}(x,y)} \right] = X(3[A-B]-2)|_{(x,y)} , \tag{139}$$

where the permutability of translation invariant symmetric two-point functions in a product, e.g.  $XYZ|_{(x,y)} = XZY|_{(x,y)} = \dots$  (cf. also eq. (136)), is crucial. Similar useful relations are easily obtained for any power of  $[A-B]$  multiplying  $G_j$  or  $G_{12}$ .

### Figure captions:

Fig. 1: Deceleration and localization of a non-relativistic parton in a gluonic environment. Shown are the unique deceleration  $v/v_0$ , eq. (66), and the relative change of the width  $(w-\beta)/\beta$ , eq. (69), of the wave packet as a function of  $t_+ \equiv f_+ t$ . We set  $\alpha_< = 0.1\alpha$ ; see eqs. (70) for the definitions of the parameters. For comparison free particle results (dashed curves) are given.

Fig. 2: Qualitative picture of Momentum Space Mode Separation: “Unobservable” gluonic environment modes are confined to the inner region II (roughly  $k_0^2 < m_\pi^2$  and  $\vec{k}^2 < \Lambda_{QCD}^2$ ). Partons live in the outer region I.

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